Tides in Oceans Bounded by Meridians. III. Ocean Bounded by Complete Meridian: Semidiurnal Tides

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TIDES IN OCEANS BOUNDED BY MERIDIANS

III. OCEAN BOUNDED BY COMPLETE MERIDIAN: SEMIDIURNAL TIDES

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1. Introduction

The first two parts of this series of memoirs have dealt with the general equations for the motion in an ocean bounded by a complete meridian (Proudman 1936) and with their application to the diurnal tide (Doodson 1936 *a*). This present memoir is concerned with the semidiurnal tide (K_2).

The investigation, as in Part II, is dependent upon the formulation and numerical solution of the equations resulting from the use of sixty-three co-ordinates or variables, and the solution has been effected and is illustrated for twenty values of the depth of the ocean, from very large to medium depths. Though a part of the work of solution is common to all cases, in effect twenty sets of equations, each set having sixty-three equations, have been solved.

The results are of exceedingly great interest in connexion with tidal theory. The genesis and development of amphidromic systems, as the depth is changed, can be fully traced. The possible systems of cotidal lines are of great variety. The type of amphidromic system changes so rapidly with the depth that a single illustration of the tides for a particular depth is of doubtful value for comparison with an actual ocean. It is hoped that the complete series of illustrations will provide material for the derivation of principles which can be applied to terrestrial oceans.

2. Notation

We shall denote by a the radius of the earth, by g the acceleration of gravity at the earth's surface, and by h the depth of the ocean, supposed uniform. Let O be the centre of the ocean, on the equator, let A be a point to the east of O, and let P be any variable point of the ocean. Then we shall denote by θ, χ the side OP and the angle AOP, respectively, of the spherical triangle AOP. Further, we shall denote by ζ the elevation of the free surface of the ocean at any time at P; by u, v the components of current at any time at P in the directions of OP and a right angle in advance of OP, respectively; by $\bar{\zeta}$ the "equilibrium-form" of ζ corresponding to the disturbing forces; and by H the maximum value of $\bar{\zeta}$.

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The motion will be taken as having a time factor $e^{i\sigma t}$ where the "speed" σ is equal to twice the angular speed of the earth's rotation, so that the semidiurnal tide considered will be the luni-solar tide K_2. In Proudman's formulae, f will thus be taken as unity.

The equations given by Proudman refer to certain Lagrangian co-ordinates denoted by p_r^n and p_{-r}^n, and to certain coefficients $\beta_{r,s}^{n,m}, \ldots$ defined in Part I. The parameter β is associated with the depth (Part I (3·93)) in the form

$$\beta = \frac{\sigma^2 a^2}{gh}. \tag{2·1}$$

3. Formulae for auxiliary functions

The solution given by Proudman makes use of two auxiliary functions, ϕ and ψ, which can be written for the semidiurnal case as

$$\phi = \sum_{r,n} p_r^n \pi_r^n F_r^n(\theta) \cos n\chi \, e^{i\sigma t}, \tag{3·1}$$

$$\psi = \sum_{r,n} p_{-r}^n \pi_r^n F_r^n(\theta) \sin n\chi \, e^{i\sigma t}, \tag{3·2}$$

where

$$\pi_r^n = \{\tfrac{1}{2}\pi r(r+1)\}^{-\frac{1}{2}} \quad (n>0), \tag{3·3}$$

$$\pi_r^0 = \{\pi r(r+1)\}^{-\frac{1}{2}}, \tag{3·4}$$

and where $F_r^n(\theta)$ is a form of the associated Legendre function (Doodson 1936*b*).

4. Formulae for the Lagrangian co-ordinates

In order to deal with real quantities throughout, and to avoid the continual writing of $i = \sqrt{-1}$, we shall deal with ip_r^n where the co-ordinates are imaginary.

The equations of Part I (3·91), (3·92), for the semidiurnal case then take the form

$$p_r^n = \Pi_r^n + \frac{\beta}{\lambda_r}\left\{p_r^n - \sum_{s,m} \beta_{r,s}^{n,m}\, ip_s^m - \sum_{t,m} \beta_{r,-t}^{n,m}\, ip_{-t}^m\right\} \quad (r, n, t \text{ even};\ s, m \text{ odd}), \tag{4·1}$$

$$ip_r^n = i\Pi_r^n + \frac{\beta}{\lambda_r}\left\{ip_r^n + \sum_{s,m} \beta_{r,s}^{n,m}\, p_s^m + \sum_{t,m} \beta_{r,-t}^{n,m}\, p_{-t}^m\right\} \quad (r, n, t \text{ odd};\ s, m \text{ even}), \tag{4·2}$$

$$p_{-r}^n = \Pi_{-r}^n + \sum_{s,m} \beta_{-r,-s}^{n,m}\, ip_{-s}^m \quad (r, m \text{ odd};\ s, n \text{ even}), \tag{4·3}$$

$$ip_{-r}^n = i\Pi_{-r}^n - \sum_{s,m} \beta_{-r,-s}^{n,m}\, p_{-s}^m \quad (r, m \text{ even};\ s, n \text{ odd}), \tag{4·4}$$

where

$$\Pi_{-r}^n = \sum_{s,m} \beta_{-r,s}^{n,m}\, ip_s^m \qquad (r, s, m \text{ odd};\ n \text{ even}), \tag{4·5}$$

$$i\Pi_{-r}^n = -\sum_{s,m} \beta_{-r,s}^{n,m} p_s^m \quad (r, s, m \text{ even};\ n \text{ odd}), \tag{4·6}$$

$$\lambda_r = r(r+1). \tag{4·7}$$

The values of $\beta^{n,m}_{r,s}$, etc. are obtained according to the relations appropriate to "the even solution" of Part I (4·41, 4·42 and 4·43), and these relations also determine the statements as to r, s, m, n, t being odd or even.

The values of Π^n_r (r even) and $i\Pi^n_r$ (r odd), from Part I (4·51), are as follows:

n	r	Π^n_r	n	r	$i\Pi^n_r$
0	2	$-0\cdot45764\ H/h$	1	1	$0\cdot76750\ H/h$
2	2	$0\cdot26422$		3	$0\cdot19539$
				5	$-0\cdot03062$
				7	$0\cdot00999$
				9	$-0\cdot00503$
				11	$0\cdot00277$

5. Transformation of principal equations

The tabular representation of the equations given above should be found in Tables I–VI, but to save space these tables are represented as slight modifications of tables appropriate to the diurnal tide.

As in Part II, 5, equation (4·3) has been transformed by substituting from (4·4) by successive approximations so as to obtain p^n_{-r} in terms of Π^m_{-s} and $i\Pi^m_{-s}$, the results being given in full in Table VII. The interpretation of this table is given by the example

$$p^2_{-3} = 1\cdot0205\,\Pi^2_{-3} + 0\cdot0033\,\Pi^2_{-5} + \ldots + 0\cdot0992\,i\Pi^1_{-2} + \ldots,$$

and all the other tables can be similarly interpreted.

In the previous part no further transformations were considered as economical of labour, and a solution was effected in series of powers of β, but such methods are not in this case available for the desired range of β owing to the increase in the size of the coefficients. It was found that the least labour was involved by eliminating p^n_{-t} from equations (4·1) to (4·4).

Hence, (4·4) was also transformed to give ip^n_{-r} in terms of Π^m_{-s} and $i\Pi^m_{-s}$, Table VIII. Then (4·5), (4·6) were used to eliminate Π^m_{-s} and $i\Pi^m_{-s}$, and so to give p^n_{-r} and ip^n_{-r} in terms of p^m_s and ip^m_s, Tables IX and X. The methods of computation and checking are exactly the same as those described in Part II.

The elimination of p^n_{-r} and ip^n_{-r} from (4·1) and (4·2) was then effected, leaving equations connecting together p^n_r and ip^n_r. The problem was thus reduced to the solution of the thirty-six simultaneous equations given in Tables XI and XII.

6. Solution for small values of β

The first method which was exploited was one on similar lines to that which was so successful for the diurnal tide, in which series of expansions in powers of $\beta/40$ were derived for certain of the principal co-ordinates. As the coefficients of the co-ordinates in the equations for the semidiurnal tide were twice as great as those for the diurnal tide it was found that a similar method could only be applied for a much smaller range of β.

It was decided to express each co-ordinate in terms of

$$H/h,\ ip_1^1,\ p_2^0,\ p_2^2,\ ip_3^1,\ ip_3^3, \tag{6·1}$$

in a series expansion in powers of $\beta/10$, and to obtain the results for $\beta = 1, 2 \dots 10$, as given in Table XIII. From this point onwards it was hoped to extrapolate values for $\beta = 11$, and to use these by substituting in the principal equations in order to get more accurate values. This method was not very easy, and it was abandoned after getting as far as $\beta = 14$. The reason for this was that the fundamental equations show the likelihood of resonance taking place for values of β approximately equal to $r(r+1)$. This difficulty had been foreseen, but it was expected that by stopping short of $r = 4$ in the list of independent variables (6·1) no real trouble would occur until β approached 20. It had been expected that it would then be possible to change the independent variables so as to exclude ip_1^1, p_2^0, p_2^2, and to include p_4^0, p_4^2, p_4^4, but no simple way of effecting this transformation was discovered.

The substitution of the results given in Table XIII in the equations for the five independent variables (6·1) in Tables XI and XII, which equations were not used in the foregoing computations, yielded five simultaneous equations in these variables, for each value of β, and these are tabulated in Table XIV. The solutions of these equations are given in Table XVII, and the resulting values of $p_r^n \pi_r^n$ and $p_{-r}^n \pi_r^n$, after substituting for the independent variables in Table XIII, applying Tables IX and X, and multiplying by π_r^n, are given in Tables XIX and XX respectively.

7. The general method of solution

The experiences outlined in the previous article showed that it would be necessary to take as independent variables all the co-ordinates with suffix r such that $r(r+1)$ was less than the largest value of β, and it was decided to express the co-ordinates in terms of

$$H/h,\ ip_1^1,\ p_2^0,\ p_2^2,\ \dots,\ \ p_6^4,\ p_6^6, \tag{7·1}$$

and to use series expansions in powers of $\beta/40$. This was done, and then the co-ordinates were tabulated by the series for $\beta = 8, 12, 16, 20, \dots, 40$, but in the sequel it was decided for the present to give the final results only for as far as $\beta = 20$. It was also found that the results for $\beta = 8$ and 12 corresponded with those obtained by the earlier method. Hence Table XV only gives the tabulations for $\beta = 12$, 16, 20 and 24. It will be seen that in this table the coefficients run so smoothly that interpolation for other values of β can easily be made.

Substitution for the dependent co-ordinates from Table XV in the previously unused equations for the independent co-ordinates in Tables XI and XII yielded equations in terms of the independent co-ordinates only, and so were obtained a set of fifteen simultaneous equations for each value of $\beta = 8, 12, 16, \dots, 40$. A selection of these is given in Table XVI.

In these tables the equations have been multiplied by $\lambda_r = r(r+1)$ in order to obtain symmetry about a diagonal, and the entries show that interpolation can be accurately effected for intermediate values of β, which was carried out for $\beta = 15, 17, 18, 19$. It has not been considered necessary to include these interpolated tables.

The solution of the equations from $\beta = 15$ to $\beta = 20$ offered no real difficulty, and the resulting values of p_r^n are given in Table XVIII.

From these values of the independent co-ordinates the dependent co-ordinates were computed by using Table XV and thence Tables IX and X. The results, after multiplying by π_r^n from (3·3) and (3·4) are given in Tables XIX and XX, which thus contain all the values of the 63 co-ordinates, tabulated for $\beta = 0, 1, \ldots, 20$.

8. Computation of ϕ, ψ, u, v

Having obtained the co-ordinates, it is a simple process to obtain from (3·1) and (3·2) the values of the auxiliary functions ϕ and ψ, which are tabulated in Tables XXI and XXII.

It may be remarked that these tables are given in series involving $\cos s\theta$ or $\sin s\theta$, and the convergence of the coefficients is very good in all cases.

The values of the components of velocity, u and v, follow from the equations (2·6) in Part I, which can be written as

$$2i \sin\theta \frac{u}{\sigma a} = 2\sin\theta \frac{\partial\phi}{\partial\theta} + 2\frac{\partial\psi}{\partial\chi}, \tag{8·1}$$

$$-2i \sin\theta \frac{v}{\sigma a} = 2\sin\theta \frac{\partial\psi}{\partial\theta} - 2\frac{\partial\phi}{\partial\chi}. \tag{8·2}$$

The differentiation of the Fourier series, whether in θ or χ, is a very simple process, and so also is the simultaneous multiplication by $2\sin\theta$.

The whole calculation was effected on a calculating machine by suitably arranging the terms of ϕ and ψ, and by placing alongside them the appropriate multiples on a strip of paper. The actual values of u and v have not been obtained other than in the forms (8·1) and (8·2), which were most convenient for the subsequent stages. It has not been thought necessary in this part to print the expansions for u and v.

9. Computation of ζ

The last stages of calculation are concerned with the evaluation of ζ, using the formula

$$\frac{\partial}{\partial\theta}\left(\frac{\zeta-\bar{\zeta}}{h}\right) = \beta\left(-i\frac{u}{\sigma a} + \sin\theta\sin\chi\frac{v}{\sigma a}\right) \tag{9·1}$$

with $$\frac{\bar{\zeta}}{h} = H\{(0·25+0·75\cos 2\theta) - (0·25-0·25\cos 2\theta)\cos 2\chi + i\sin 2\theta\cos\chi\}e^{i\sigma t} \tag{9·2}$$

from Part I (2·2) and (2·41). It was shown in Part II, 12, that the use of (9·1) gives much greater convergence than either of two other possible methods for determining ζ.

The processes involved were sufficiently explained in Part II, 12, and the results are given in Table XXIII. As in the diurnal case, odd and even values of s occur in the expansions in $\cos s\theta$ and $\sin s\theta$, and the cases where $s = 0$ need special reference. A constant of integration is introduced to satisfy (1) the condition of invariability of volume or (2) the condition that the elevation at the point $\theta = 0$, $\chi = 0$ shall be independent of χ. The first condition is satisfied automatically by the factors in χ except when $n = 0$, and in this case it is necessary to take the constant equal to

$$-\int_0^{\frac{1}{2}\pi} \zeta_0 \sin\theta \, d\theta, \tag{9·3}$$

where ζ_0 is that part of ζ with $n = 0$. The second condition requires $\zeta_n = 0$, when $\theta = 0$, $n > 0$, where ζ_n is the part of ζ with factor $\cos n\chi$.

No constant of integration is required for terms arising from $\sin s\theta$, but it should be noted that the term against $s = 0$ has a coefficient of $\theta \cos n\chi$. The significance of this was pointed out in Part II, 12.

From the values of ζ in the complex form we deduce the form

$$\zeta = \zeta_1 \cos\sigma t + \zeta_2 \sin\sigma t, \tag{9·4}$$

and thence

$$\zeta = R\cos(\sigma t - \gamma), \tag{9·5}$$

where γ is the lag of phase of the semidiurnal tide behind the phase of the semidiurnal equilibrium tide (K_2) on the central meridian.

The values of ζ_1/H and ζ_2/H are given in Table XXIV, but they were computed for intervals of 10° in θ and χ. The tables have been contracted to save space, and for the same reason the numerical values of R, γ are not given, as it is considered that the charts give these to sufficient accuracy for all essential purposes. For the final discussion of the results the values of ζ_1 and ζ_2 will probably be more useful than R and γ.

The values of R and γ are given on the charts for the integral values of β from 0 to 20. Only a quadrant of the ocean is shown, because of symmetry or asymmetry about the equator and central meridian. The charts are drawn without respect to systems of projections, as though θ, χ were two-dimensional polar co-ordinates.

The co-range lines are drawn on a uniform plan, for ranges equal to the maximum range multiplied by 0·8, 0·6, 0·4 and 0·2. The factor to give the range in terms of the maximum equilibrium elevation H is given for each chart. The depths in miles are also given.

Before leaving the description of the computation of ζ, it is of interest to quote the results of certain tests which have been made on the whole of the calculations. One of the formulae quoted in Part II, 12, for the computation of ζ is the expression

$$\frac{\zeta}{h} = -\Sigma\lambda_r p_r^n \pi_r^n F_r^n(\theta) \cos n\chi \, e^{i\sigma t} \tag{9·6}$$

derived from Part I (3·51). The convergence shown by the terms of the formula is necessarily very poor because of the factor $\lambda_r = r(r+1)$, but the formula involves the fundamentals of the method to such a degree that any values of ζ obtained by its use will serve to confirm the general accuracy of the values obtained by the standard method.

For $\beta = 20$, the coefficients of $-\cos n\chi e^{i\sigma t}$ for three values of θ are given under (*b*) in the following table, while the values for the standard method are given under (*a*):

	$\theta = 0°$		$\theta = 40°$		$\theta = 90°$	
n	(*a*)	(*b*)	(*a*)	(*b*)	(*a*)	(*b*)
0	2·15	2·12	−1·32	−1·32	1·26	1·30
2			−3·05	−3·03	2·06	2·14
4			0·02	−0·09	3·62	3·26
6			0·06	0·04	−0·41	−0·21
1			1·08	1·05	1·42	1·19
3			−0·35	−0·38	1·34	1·13
5			−0·14	−0·05	−2·08	−1·59
7			−0·01		−0·04	

As was pointed out in Part II, 12, the differences between (*a*) and (*b*) would be anticipated to be of an oscillatory nature, so that (*a*) might be considered as a smoothed version of (*b*). It was also pointed out that the differences between the results from the two methods of computing ζ will be greatest at the boundary ($\theta = 90°$).

The above comparisons can be regarded as highly satisfactory, and the same is true of comparisons which have been made for other values of β.

10. The critical depths

The critical depths are those at which resonance tends to take place, and their values have been obtained by interpolation in the end equations during the process of solving the differential equations. (Unless all details of the computations were given it would not be of any value to give these end equations.) By these methods of computation the critical values of β, and the corresponding depths of the ocean, are as follows:

β	h
1·896	153,200 ft. = 29·0 miles
5·87	49,500 ,, = 9·37 ,,
9·41	30,900 ,, = 5·85 ,,
12·69	22,880 ,, = 4·33 ,,
15·62	18,590 ,, = 3·52 ,,

These can be confirmed by graphical (or even linear) interpolation for zero values of $1/(\zeta_1 - \bar{\zeta}_1)$ from the following table of $\zeta_1 - \bar{\zeta}_1$ at the pole:

β	$\zeta_1 - \bar{\zeta}_1$	β	$\zeta_1 - \bar{\zeta}_1$	β	$\zeta_1 - \bar{\zeta}_1$
0	0·00	7	− 4·99	14	− 5·69
1	0·38	8	− 5·42	15	−10·25
2	− 5·75	9	−14·75	16	14·57
3	− 0·58	10	9·35	17	4·06
4	0·04	11	4·65	18	2·62
5	1·64	12	6·22	19	2·43
6	−22·95	13	−14·10	20	3·23

Other sequences of $\zeta_1 - \bar{\zeta}_1$ or $\zeta_2 - \bar{\zeta}_2$ could also be used in confirmation.

The values of ϕ, ψ, u, v, ζ, R and γ have all been computed, and diagrams prepared, for these critical depths, but they are not published because the results have no outstanding points of interest. They will be referred to in the remarks on the charts.

11. REMARKS ON THE CHARTS FOR $\beta = 0$ TO $\beta = 2$

Beginning with the case of the equilibrium tide for the infinite depth corresponding to $\beta = 0$, we note that as β increases the pole is no longer an amphidromic point and that the cotidal lines swing out to be spaced terminally along the bounding meridian. A minimum range develops also at a point on the central meridian.

The changes between $\beta = 1$ and $\beta = 2$ are not easily traced, owing to the great changes in the depths, but the development of the amphidromic system can be investigated as follows.

When $\beta = 0$, then

$$\bar{\zeta}_1/H = (0{\cdot}75\cos 2\theta + 0{\cdot}25) + (0{\cdot}25\cos 2\theta - 0{\cdot}25)\cos 2\chi, \tag{11·1}$$

and when $\beta = 1{\cdot}896$, ζ_1 has been computed to be approximately proportional to

$$[\zeta_1]/H = (-4{\cdot}6\cos\theta + 2{\cdot}3) + (4{\cdot}6\cos\theta - 4{\cdot}6)\cos 2\chi, \tag{11·2}$$

but of course it is then infinitely large.

When $\beta = 1$ we have ζ_1 practically equal to

$$\bar{\zeta}_1 + 0{\cdot}08\,[\zeta_1]. \tag{11·3}$$

This suggests that for our purposes we can write, more generally

$$\zeta_1 = \bar{\zeta}_1 + x[\zeta_1], \tag{11·4}$$

where x is a variable whose only interest is that it steadily increases as β increases from $\beta = 0$ to $\beta = 1{\cdot}896$. On the central meridian this gives

$$\zeta_1/H = \cos^2\theta - (9{\cdot}2\cos\theta - 6{\cdot}9)\,x. \tag{11·5}$$

Since ζ_2 is always zero on this meridian, then an amphidromic point will be found on the central meridian when ζ_1 is also zero, and this occurs when

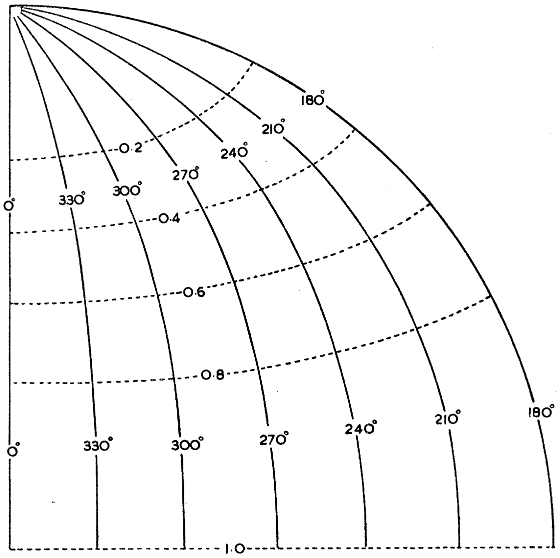
$$x = \frac{\cos^2\theta}{9{\cdot}2\cos\theta - 6{\cdot}9}. \tag{11·6}$$

When x is infinite, then an amphidromic point is found at the point given by

$$\cos\theta = 6{\cdot}9/9{\cdot}2 = 0{\cdot}75. \tag{11·7}$$

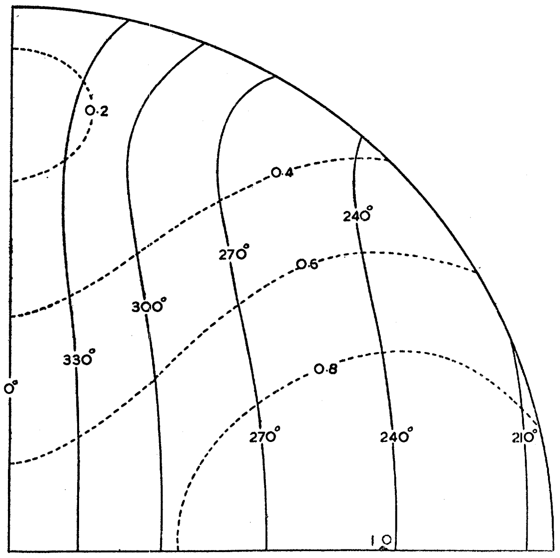
This is in agreement with the results obtained from the actual chart, which is also in close agreement with the chart for $\beta = 2$.

We also note that $$\frac{\partial}{\partial\theta}\cdot\frac{\zeta_1}{H} = -(2\cos\theta - 4{\cdot}6x)\sin\theta, \tag{11·8}$$



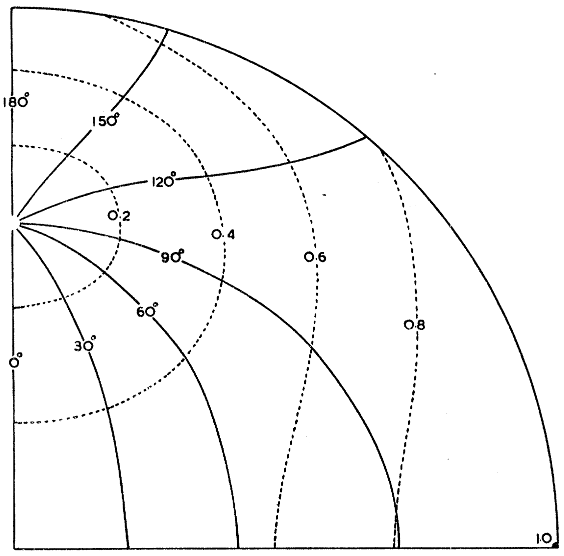
$\beta = 0$, depth infinite, factor $= H$

FIG. 1



$\beta = 1$, depth $= 55{\cdot}0$ miles, factor $= 1{\cdot}52\,H$

FIG. 2



$\beta = 2$, depth $= 27{\cdot}5$ miles, factor $= 9{\cdot}85\,H$

FIG. 3

and therefore ζ_1 is a minimum when $x = 0{\cdot}22\cos\theta$ and a maximum when $\theta = 0$, if x is less than 0·22, but when x is greater than 0·22 the minimum value occurs with $\theta = 0$. Hence, as x increases from zero, the point of minimum range on the central meridian moves from the pole towards the equator, and stays there until the range becomes zero, when the first amphidromic system develops and thereafter the amphidromic point moves towards the pole, as may be seen from (11·6), for x increases as $\cos\theta$ diminishes from unity. After the first critical depth is reached, x becomes a large negative quantity and the movement of the amphidromic point continues towards the pole.

It should also be noted that instead of the equator being a line of maximum range (as for $\beta = 0$), we get a point of maximum range on the equator, and that this moves outwards (see the chart for $\beta = 1$) to the extremity of the equator.

12. REMARKS ON THE CHARTS FOR $\beta = 2$ TO $\beta = 6$

The changes between $\beta = 2$ and $\beta = 3$ are also difficult to follow but as the amphidromic system about the point on the central meridian still has a rotation in the positive sense, as in the case of $\beta = 2$, it appears as though the second amphidromic system near the equator has its origin in the crowding together of the cotidal lines on the equator.

Between $\beta = 3$ and $\beta = 4$ the amphidromic point on the central meridian has gone out at the pole, the other point has travelled north also and has vanished at the boundary, as is shown by the progression of cotidal lines along the equator being the same in both cases.

The cotidal system for $\beta = 4$ is very interesting, for an approach to a nodal line is shown in the closeness of the parallel cotidal lines. Wherever this occurs, very rapid changes in the apparent character of the cotidal systems may be anticipated, as is seen in the chart for $\beta = 5$.

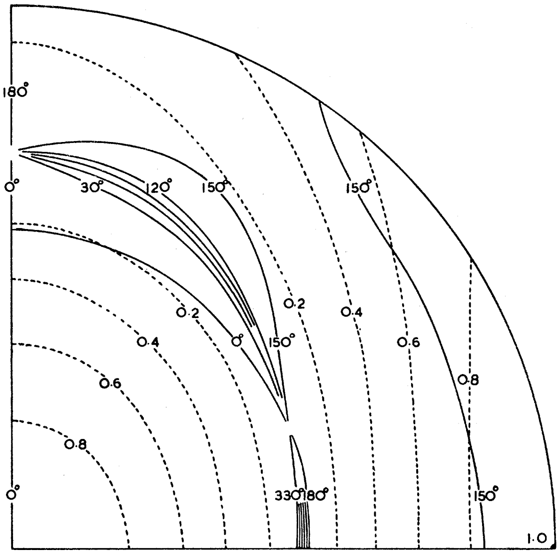
Between $\beta = 5$ and $\beta = 6$ there is obviously a critical value, as the rotation of the amphidromic system is in the same sense in each, though similarly situated lines differ in phase by 180°.

The amphidromic system for the critical value $\beta = 5{\cdot}87$ is practically identical with that for $\beta = 6$.

Note that the point of maximum range is at the junction of the bounding meridian and the equator.

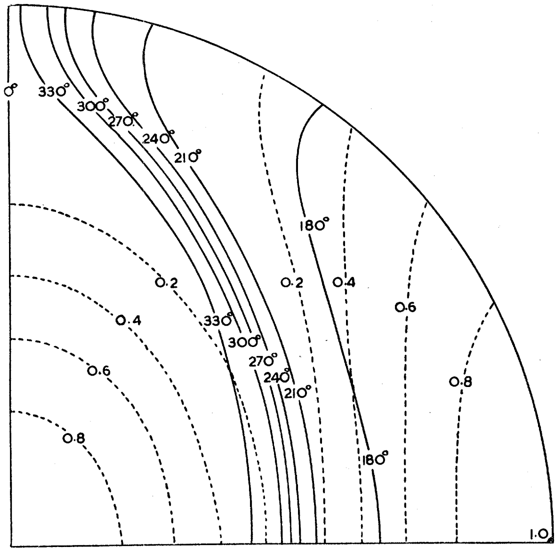
13. REMARKS ON THE CHARTS FOR $\beta = 6$ TO $\beta = 10$

The chart for $\beta = 6$ shows a point of minimum range on the central meridian, and this can be taken to indicate the imminent development of an amphidromic system there, as for $\beta = 7$. The rotation of the amphidromic system in the centre of the quadrant for $\beta = 7$ is the same as that for $\beta = 6$, which shows continuity in the region.



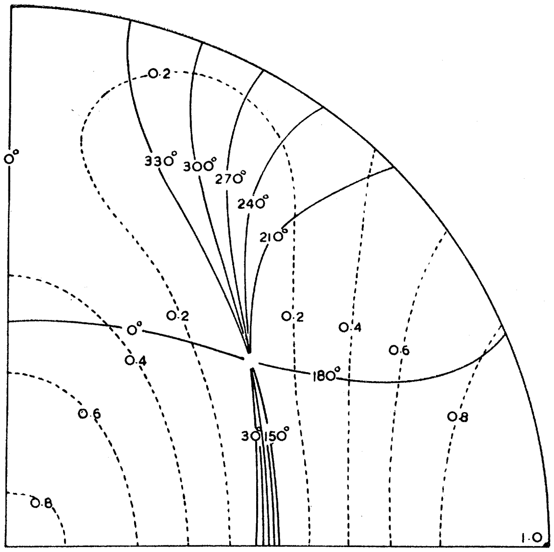
$\beta = 3$, depth = 27·5 miles, factor = $9{\cdot}85H$

Fig. 4



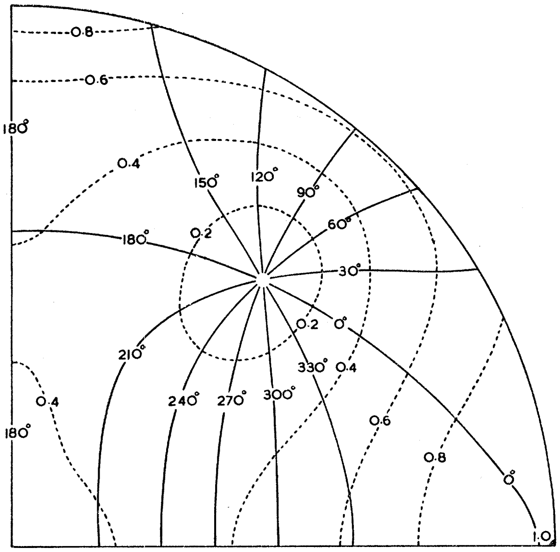
$\beta = 4$, depth = 13·75 miles, factor = $2{\cdot}69H$

Fig. 5



$\beta = 5$, depth = 11·00 miles, factor = $4{\cdot}92H$

Fig. 6



$\beta = 6$, depth = 9·17 miles, factor = $26{\cdot}6H$

Fig. 7

Apart from a movement of the amphidromic system in the quadrant towards the boundary there is little change in the general characteristics of the charts for $\beta = 7$ and $\beta = 8$, but the crowding of the lines for 210–270° shows the approach to another nodal system, which leads to the generation of a new amphidromic system in the centre of the quadrant for $\beta = 9$, together with the reversal of rotation of the system previously existing round a point on the central meridian.

Proudman has suggested to the author that these changes may be followed more easily by considering only the lines of zero elevation for the two phases for ζ_1 and ζ_2. This suggestion is of great value in such cases as the above. A small movement of either line will completely reverse the apparent rotation of the amphidromic system when the two lines are nearly parallel. In a later memoir this suggestion will probably be developed more fully.

Between $\beta = 9$ and $\beta = 10$ there is a critical value, evidenced by the increment of 180° for similarly situated lines, otherwise the directions of the rotation are the same, and the lines have spread out. The intrusion of a new system at the boundary is to be anticipated for the curves for $\beta = 9$.

The chart for the critical depth for $\beta = 9{\cdot}41$ is similar to that for $\beta = 10$ except for the new system at the boundary, though the lines are more evenly spaced along the equator, and the two amphidromic points are closer together.

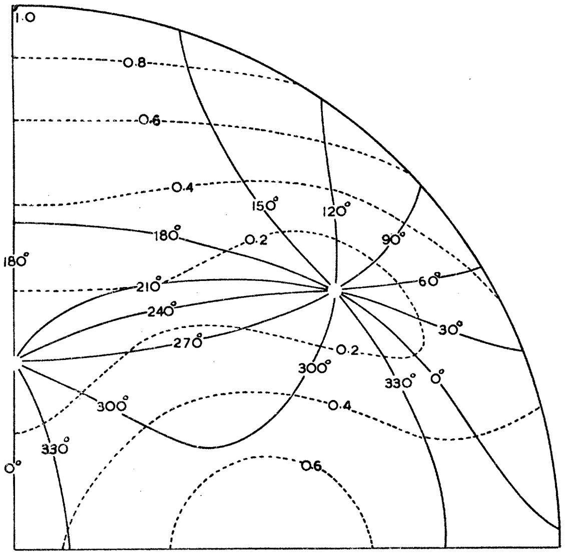
Note that for $\beta = 7$ the point of maximum range now occurs at the pole, where it also occurs for $\beta = 8$, but the development of the maximum at the centre of the ocean is to be seen completed for $\beta = 9$.

14. Remarks on the charts for $\beta = 10$ to $\beta = 20$

At first sight there does not seem to be much association between the systems for $\beta = 10$ and those for $\beta = 11$, but a consideration of the directions of rotation shows that the amphidromic point near the boundary in the former case has moved inwards, while that on the central meridian has remained nearly stationary, so that the lower amphidromic system for $\beta = 10$ has probably vanished at the equator.

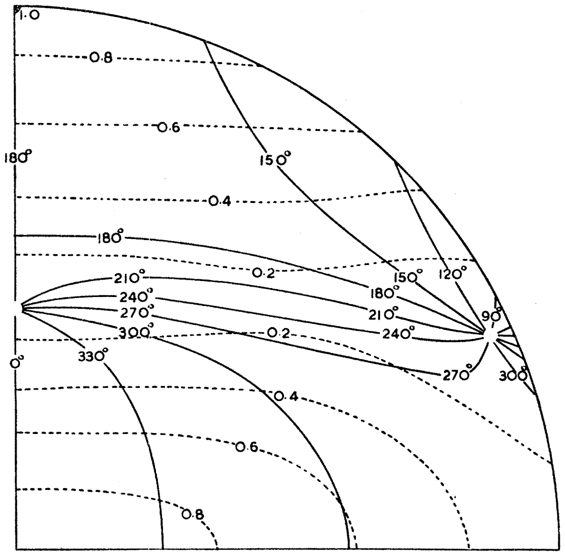
Note the changes in the position of the point of maximum range between $\beta = 10$ and $\beta = 11$.

The changes from $\beta = 11$ to $\beta = 12$ are small, but another fundamental change takes place between $\beta = 12$ and $\beta = 13$. The crowding together near the central meridian of the lines for 180, 210 and 240° for $\beta = 12$ indicates an approach to a nodal system, facilitating the development of another amphidromic point. The increment of 180° in phase for similarly situated lines for $\beta = 12$ and $\beta = 13$ is a consequence of passing through the critical depth. The directions of rotation of the upper system and the system in the middle of the quadrant are the same for both depths, thus verifying that it is the lower system on the central meridian which has crept in. The chart for the critical value $\beta = 12{\cdot}69$ is very like that for $\beta = 13$.



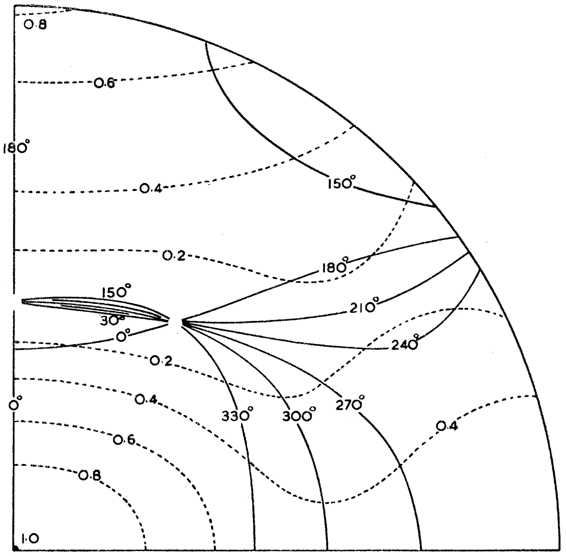
$\beta=7$, depth = 7·86 miles, factor = $4{\cdot}99H$

FIG. 8



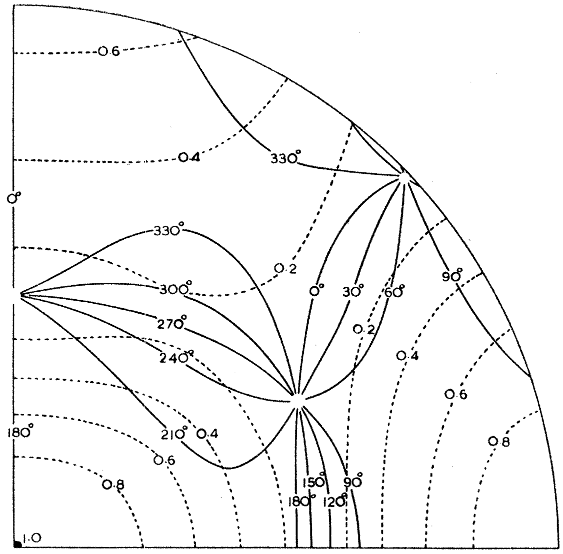
$\beta=8$, depth = 6·88 miles, factor = $5{\cdot}42H$

FIG. 9



$\beta = 9$, depth = 6·11 miles, factor = $18{\cdot}3H$

FIG. 10



$\beta = 10$, depth = 5·50 miles, factor = $12{\cdot}71H$

FIG. 11

Note the position of the point of maximum range for $\beta = 13, 14, 15$.

Between $\beta = 13$ and $\beta = 14$ the changes are in the direction of an approach to a nodal system parallel to the equator, which is favourable to the reversal of rotation in any amphidromic system. This is more evident in the chart for $\beta = 15$ in which the directions of rotations for the two lower amphidromic systems are the reverse of those for $\beta = 14$.

The charts for $\beta = 15$ and $\beta = 16$ do not show any great change, apart from the change of 180° due to passing through the critical value, and the chart for the critical value $\beta = 15{\cdot}62$ does not differ much from either of those for $\beta = 15$ and $\beta = 16$. For $\beta = 17$ the only significant change is towards the development of a new system near the boundary, which is seen developed for $\beta = 18$. A change near the central meridian is of interest, but very little light can be shed on the mechanism of the change, except that it would appear as though both points had first moved together, as is indeed suggested by a comparison of positions for $\beta = 16$ and $\beta = 17$, and that the resulting system had then moved away from the central meridian. Probably a consideration of the zero-lines of ζ_1 and ζ_2 will throw light on this movement.

Between $\beta = 18$ and $\beta = 19$ the lowest amphidromic point for $\beta = 18$ appears to go out at the equator, leaving two systems only in the quadrant, and these two systems become more evenly spaced for $\beta = 20$.

It is noticeable that while there is in general an increase in the number of amphidromic systems as β increases, yet this is not a simple function of β, as is seen from the apparent simplification for the systems from $\beta = 18$ to $\beta = 19$ or $\beta = 20$. It will probably be found that the complexity of some of the cotidal systems is due to the merely incidental approach of the zero-lines in ζ_1 and ζ_2.

The smallest depth for which illustrations have been provided ($\beta = 20$, depth $=$ 14,520 ft.) is about the average depth of the Pacific Ocean, but it is clear, in view of the rapid changes in the tidal charts with changes in depth, that none of these charts can be directly applied to an actual ocean. Such is not their function, but it is hoped that a study of the charts in a further memoir will yield principles which can ultimately be confidently applied to the tides in such an ocean as the Pacific.

ACKNOWLEDGEMENTS

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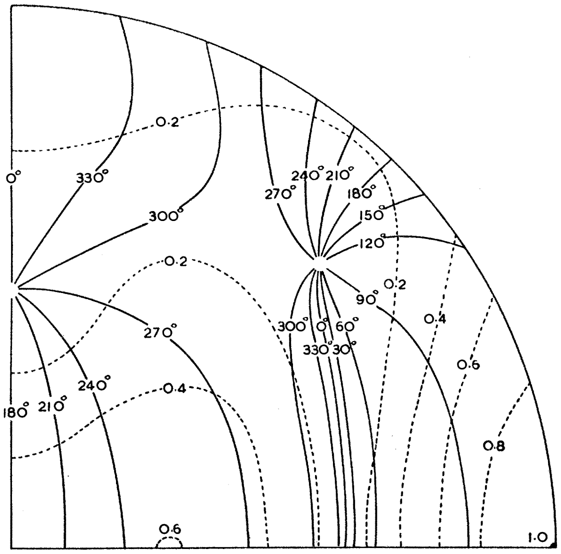
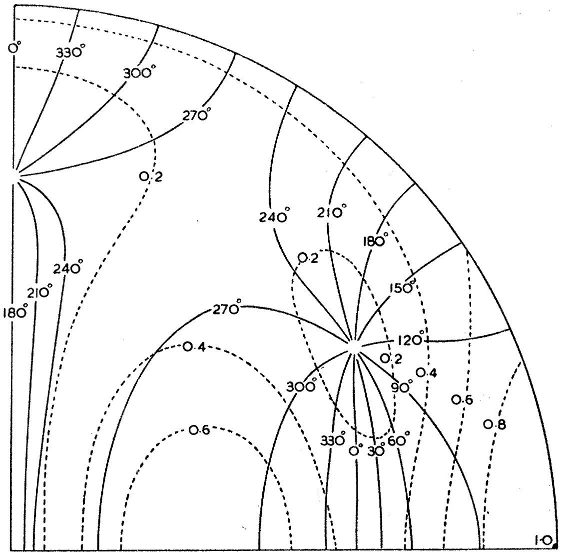


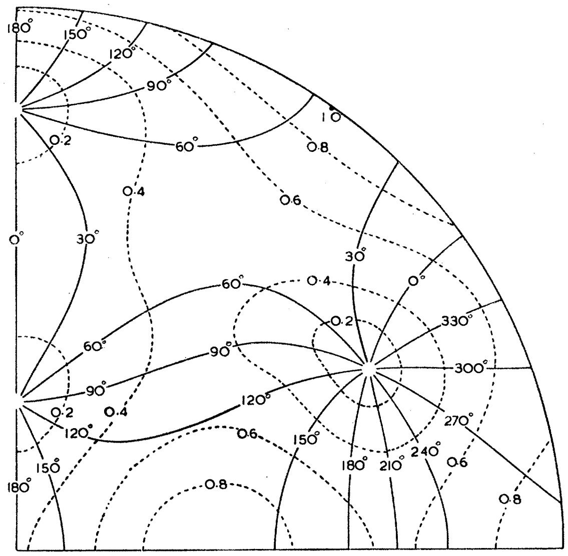
$\beta = 11$, depth = 5·00 miles, factor = 8·63H

Fig. 12



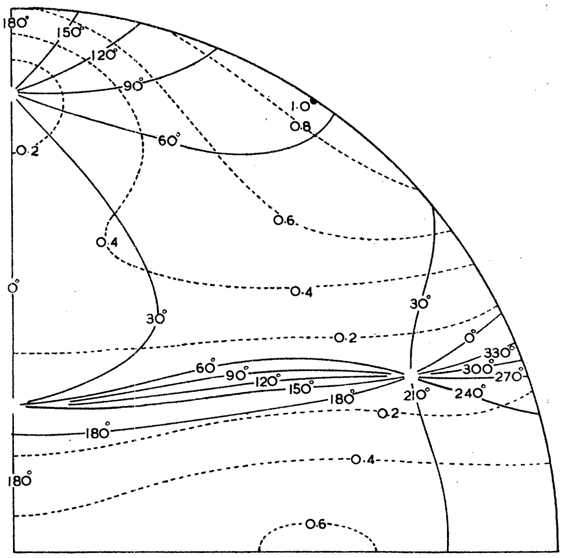
$\beta = 12$, depth = 4·58 miles, factor = 13·75H

Fig. 13



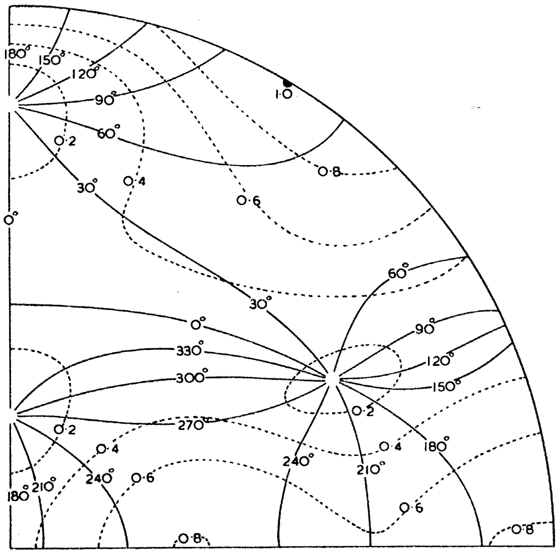
$\beta = 13$, depth = 4·23 miles, factor = 22·6H

Fig. 14



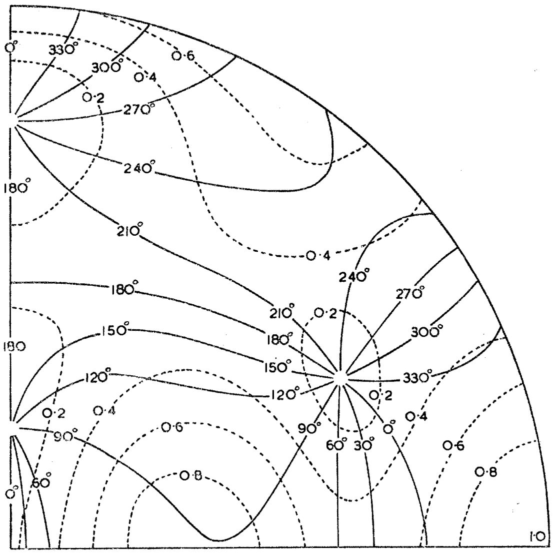
$\beta = 14$, depth = 3·93 miles, factor = 9·7H

Fig. 15



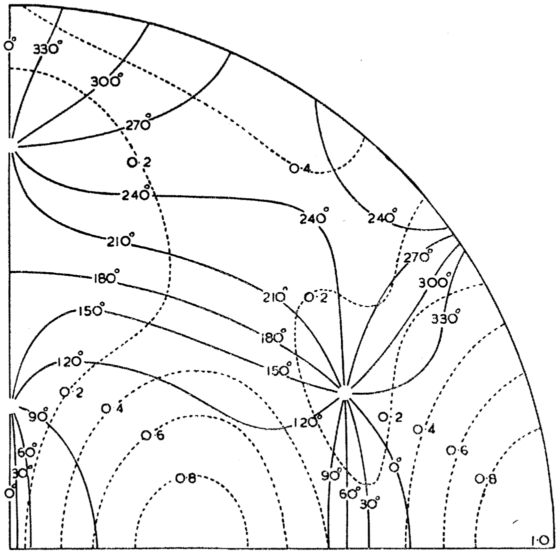
$\beta = 15$, depth = 3·67 miles, factor = 15·6H

FIG. 16



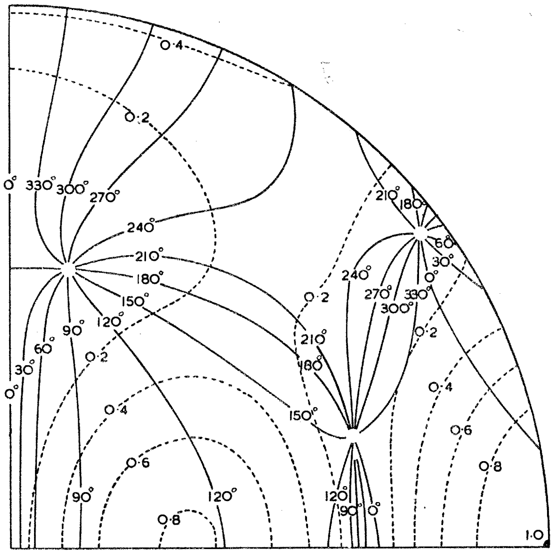
$\beta = 16$, depth = 3·44 miles, factor = 25·9H

FIG. 17



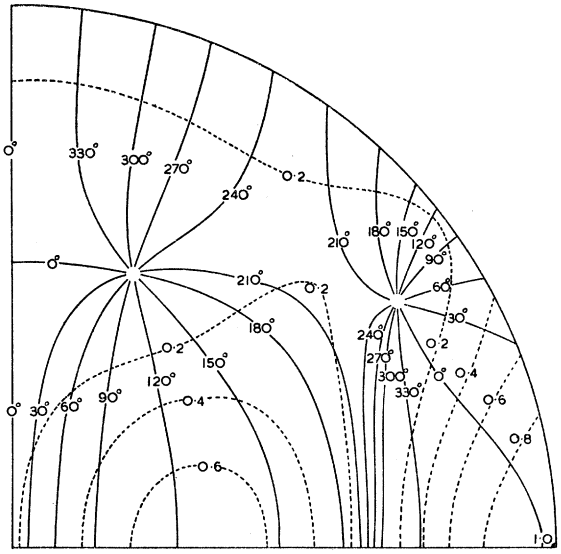
$\beta = 17$, depth = 3·24 miles, factor = 8·54H

FIG. 18



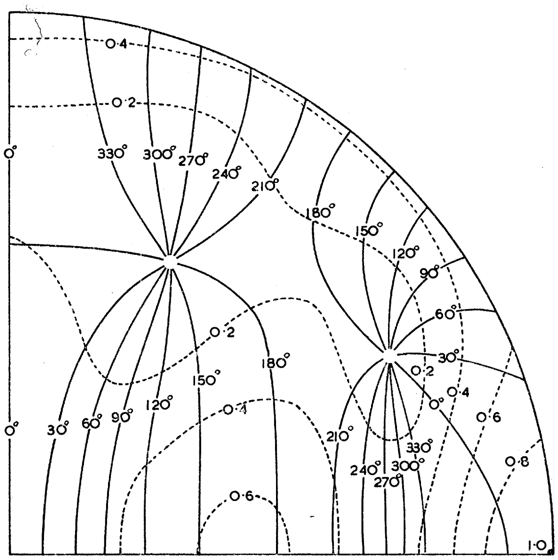
$\beta = 18$, depth = 3·06 miles, factor = 6·03H

FIG. 19



$\beta = 19$, depth $= 2{\cdot}89$ miles, factor $= 5{\cdot}57\,H$

FIG. 20



$\beta = 20$, depth $= 2{\cdot}75$ miles, factor $= 6{\cdot}57\,H$

FIG. 21

TABLE I. $p^n_{-r} = \Pi^n_{-r} + \sum_{s,m} \beta^{n,m}_{-r,-s}\, ip^m_{-s}$

The table of coefficients of ip^m_{-s} may be derived from Table V for the diurnal case, by multiplying the entries by $-\frac{1}{2}$, and omitting the group of coefficients for $n = 0$.

TABLE II. $ip^n_{-r} = i\Pi^n_{-r} - \sum_{s,m} \beta^{n,m}_{-r,-s}\, p^m_{-s}$

The table of coefficients of p^m_{-s} may be derived from the coefficients of $i\Pi^m_{-s}$ in Table I for the diurnal case, by multiplying the entries by $-\frac{1}{2}$, and omitting the group of coefficients for $m = 0$.

TABLE III. $\Pi^n_{-r} = \sum_{s,m} \beta^{n,m}_{-r,s}\, ip^m_s$

The table of coefficients of ip^m_s may be derived from Table III for the diurnal case, by multiplying the entries by $\frac{1}{2}$, and omitting the group of coefficients for $n = 0$.

TABLE IV. $i\Pi^n_{-r} = -\sum_{s,m} \beta^{n,m}_{-r,s}\, p^m_s$

The table of coefficients of p^m_s may be derived from Table II for the diurnal case, by multiplying the entries by $\frac{1}{2}$, and by adding the following group:

		$n=1$					
m	s	$r=2$	4	6	8	10	12
0	2	1804	2421	−691	353	−219	150
	4	−3315	1001	2672	−859	471	−305
	6	1636	−3227	693	2772	−944	536
	8	−1152	1510	−3176	530	2825	−994
	10	901	−1056	1440	−3143	429	2859
	12	−743	828	−998	1395	−3121	360

TABLE V. $p^n_r = \Pi^n_r + \frac{\beta}{\lambda_r}\left\{p^n_r + \sum_{s,m}(-\beta^{n,m}_{r,s})\, ip^m_s + \sum_{s,m}(-\beta^{n,m}_{r,-s})\, ip^m_{-s}\right\}$

The table of coefficients of $(\beta/\lambda_r)\, ip^m_s$ may be obtained from the entries in the upper half of Table VII for the diurnal case, by changing the sign, and by adding the following group:

		$n=0$					
m	s	$r=2$	4	6	8	10	12
1	1	4193	−1712	1118	−837	671	−560
	3	1067	1471	−569	373	−282	229
	5	−167	737	892	−334	217	−164
	7	59	−150	558	640	−234	151
	9	−28	61	−128	448	498	−180
	11	15	−31	56	−111	374	408

The table of coefficients of $(\beta/\lambda_r)\, ip^m_{-s}$ is identical with the lower half of Table VII for the diurnal case, with the following additional group:

		$n=0$					
m	s	$r=2$	4	6	8	10	12
1	2	−1804	3315	−1636	1152	−901	743
	4	−2421	−1001	3227	−1510	1056	−828
	6	691	−2672	−693	3176	−1440	998
	8	−353	859	−2772	−530	3143	−1395
	10	219	−471	944	−2825	−429	3121
	12	−150	305	−536	994	−2859	−360

TABLE VI. $ip^n_r = i\Pi^n_r + \frac{\beta}{\lambda_r}\left\{ip^n_r + \sum_{s,m}\beta^{n,m}_{r,s}\, p^m_s + \sum_{s,m}\beta^{n,m}_{r,-s}\, p^m_{-s}\right\}$

The table of coefficients of $(\beta/\lambda_r)\, p^m_s$ may be obtained from the entries in the upper half of Table VI for the diurnal case, by changing the sign, and by adding the following group:

		$n=1$					
m	s	$r=1$	3	5	7	9	11
0	2	4193	1067	−167	59	−28	15
	4	−1712	1471	737	−150	61	−31
	6	1118	−569	892	558	−128	56
	8	−837	373	−334	640	448	−111
	10	671	−282	217	−234	498	374
	12	−560	229	−164	151	−180	408

The table of coefficients of $(\beta/\lambda_r)\, p^m_{-s}$ is identical with the lower half of Table VI for the diurnal case, with the omission of the group for $m=0$.

TABLE VII. p^n_{-r} IN TERMS OF Π^m_{-s} AND $i\Pi^m_{-s}$

(Coefficients to 4 places of decimals)

Coefficients of Π^m_{-s}

		$n=2$					$n=4$				$n=6$		
m	s	$r=3$	5	7	9	11	5	7	9	11	7	9	11
2	3	1·0205	33	−18	12	−8	−22	12	−8	6			
	5	33	1·0132	8	−5	4	−49	−5	4	−3			
	7	−18	8	1·0079	3	−3	−5	−34	−2	2			
	9	12	−5	3	1·0024	3	3	−2	−23	−2			
	11	−8	4	−3	3	1·0034	−3	2	−2	−16			
4	5	−22	−49	−5	3	−3	1·0058	12	−8	6	−7	5	−4
	7	12	−5	−34	−2	2	12	1·0056	5	−4	−20	−3	2
	9	−8	4	−2	−23	−2	−8	5	1·0019	4	−3	−18	−2
	11	6	−3	2	−2	−16	6	−4	4	1·0030	2	−2	−14
6	7						−7	−20	−3	2	1·0024	6	−5
	9						5	−3	−18	−2	6	1·0012	4
	11						−4	2	−2	−14	−5	4	1·0024

Coefficients of $i\Pi^m_{-s}$

		$n=2$					$n=4$				$n=6$		
m	s	$r=3$	5	7	9	11	5	7	9	11	7	9	11
1	2	992	−255	123	−73	49	−1	1	−1				
	4	752	627	−176	91	−56	−4	1	−1				
	6	−199	522	451	−132	71	−2	−2					
	8	100	−149	395	352	−106		−1	−1				
	10	−61	78	−117	316	288				−1			
	12	42	−50	63	−96	264							
3	4	−666	−555	155	−80	50	477	−145	76	−48			
	6	189	−498	−429	125	−68	428	402	−121	65	−1		
	8	−96	145	−384	−343	103	−125	360	331	−101	−1		
	10	59	−77	115	−311	−284	66	−108	300	277			
	12	−40	48	−62	94	−261	−42	58	−91	255			
5	6		2	1			−365	−344	104	−56	291	−95	53
	8		−1	1	1		116	−335	−307	92	283	283	−89
	10				1		−63	103	−288	−271	−87	265	252
	12						41	−56	88	−247	48	−82	235

TABLE VIII. ip^n_{-r} IN TERMS OF $i\Pi^m_{-s}$ AND Π^m_{-s}

(Coefficients to 4 places of decimals)

Coefficients of $i\Pi^m_{-s}$

		$n=1$						$n=3$					$n=5$			
m	s	$r=2$	4	6	8	10	12	4	6	8	10	12	6	8	10	12
1	2	1·0106	54	−25	15	−10	8	−48	24	−15	10	−8				
	4	54	1·0098	8	−5	4	−3	−87	−8	5	−4	3				
	6	−25	8	1·0054	3	−2	3	−7	−51	−3	2	−3				
	8	15	−5	3	1·0032	2	−3	4	−3	−31	−2	3				
	10	−10	4	−2	2	1·0021	3	−3	2	−2	−20	−3				
	12	8	−3	3	−3	3	1·0009	3	−2	2	−3	−9				
3	4	−48	−87	−7	4	−3	3	1·0102	20	−12	9	−8	−11	7	−5	5
	6	24	−8	−51	−3	2	−2	20	1·0084	7	−5	6	−30	−4	3	−3
	8	−15	5	−3	−31	−2	2	−12	7	1·0056	4	−5	−4	−24	−2	3
	10	10	−4	2	−2	−20	−3	9	−5	4	1·0038	6	3	−2	−17	−3
	12	−8	3	−3	3	−3	−9	−8	6	−5	6	1·0016	−3	3	−3	−8
5	6							−11	−30	−4	3	−3	1·0036	9	−6	6
	8							7	−4	−24	−2	3	9	1·0039	5	−6
	10							−5	3	−2	−17	−3	−6	5	1·0031	7
	12							5	−3	3	−3	−8	6	−6	7	1·0014

Coefficients of Π^m_{-s}

		$n=1$						$n=3$					$n=5$			
m	s	$r=2$	4	6	8	10	12	4	6	8	10	12	6	8	10	12
2	3	992	752	−199	100	−61	42	−666	189	−96	59	−40				
	5	−255	627	522	−149	78	−50	−555	−498	145	−77	48	2	−1		
	7	123	−176	451	395	−117	63	155	−429	−384	115	−62	1	1		
	9	−73	91	−132	352	316	−96	−80	125	−343	−311	94		1	1	
	11	49	−56	71	−106	288	264	50	−68	103	−284	−261				
4	5	−1	−4	−2				477	428	−125	66	−42	−365	116	−63	41
	7	1	1	−2	−1			−145	402	360	−108	58	−344	−335	103	−56
	9	−1	−1		−1			76	−121	331	300	−91	104	−307	−288	88
	11					−1		−48	65	−101	277	255	−56	92	−271	−247
6	7								−1	−1			291	283	−87	48
	9												−95	283	265	−82
	11												53	−89	252	235

TABLE IX. p^n_{-r} IN TERMS OF p^m_s AND ip^m_s

(Coefficients to 4 places of decimals)

		$n=2$					$n=4$				$n=6$		
m	s	$r=3$	5	7	9	11	5	7	9	11	7	9	11
Coefficients of p^m_s													
0	2	380	62	−34	22	−16	−1	1	0	1			
	4	−319	305	21	−14	12	−1	−1					
	6	−58	−259	232	12	−11	1	−1					
	8	46	−23	−202	182	13		1					
	10	−41	22	−17	−160	142							
	12	43	−28	25	−27	−111							
2	2	−220	−36	20	−13	9	90	−54	35	−25			
	4	−454	−115	−21	13	−10	205	35	−24	17			
	6	72	−187	−70	−11	7	−58	174	19	−16			
	8	−57	24	−99	−46	−6	−35	−79	140	15			
	10	47	−20	11	−61	−32	30	−21	−75	110			
	12	−40	18	−10	7	−44	−32	26	−25	−52			
4	4	194	62	−65	42	−30	−187	−8	6	−5	58	−42	32
	6	146	264	−43	−23	18	−268	−135	−7	6	148	28	−22
	8	−103	45	202	−59	−15	18	−166	−93	−7	−19	138	20
	10	82	−37	23	154	−56	−16	10	−109	−66	−26	−37	113
	12	−72	37	−26	23	111	15	−10	9	−80	29	−29	−27
6	6						129	47	−49	36	−40	45	−34
	8						72	183	−15	−24	−155	14	23
	10						−54	35	156	−27	−29	−143	27
	12						48	−34	28	119	28	−26	−112
Coefficients of ip^m_s													
1	1	−2308	594	−287	170	−114	2	−2	2	−1			
	3	−1700	−2149	637	−334	212	14	−3	1	−1			
	5	2029	−1128	−2133	685	−380	2	10	−3	2			
	7	−1020	2099	−837	−2121	719	−8	1	7	−2			
	9	722	−1003	2113	−663	−2125	3	−6	1	5			
	11	−567	703	−976	2112	−549	−1	2	−4	0			
3	3	1997	−1165	465	−266	175	−3057	931	−492	313	0	−1	1
	5	2995	1202	−1458	563	−331	−2101	−2538	794	−438	6	−2	1
	7	−1187	2757	867	−1611	624	1616	−1632	−2401	786	3	5	−2
	9	788	−1118	2612	677	−1713	−839	1804	−1308	−2341	−3	2	4
	11	−602	749	−1066	2514	559	598	−889	1881	−1091	1	−3	1
5	5	−8	−12	3	0	−1	2752	−1302	609	−377	−3335	1092	−610
	7	−3	−16	−7	4	−1	2909	1790	−1403	607	−2289	−2746	892
	9	4	5	−9	−4	3	−1064	2811	1376	−1529	1425	−1915	−2583
	11	−4	−2	3	−6	−3	685	−1072	2694	1128	−749	1629	−1615

TABLE X. ip^n_{-r} IN TERMS OF p^m_s AND ip^m_s

(Coefficients to 4 places of decimals)

		$n=1$						$n=3$					$n=5$			
m	s	$r=2$	4	6	8	10	12	4	6	8	10	12	6	8	10	12
Coefficients of p^m_s																
0	2	1839	2454	-697	355	-220	150	-28	6	-2	1	0				
	4	-3353	996	2695	-866	475	-307	4	-22	7	-4	2				
	6	1640	-3252	691	2785	-948	538	21	2	-12	4	-2				
	8	-1151	1517	-3190	529	2833	-997	-6	13	1	-8	3				
	10	898	-1059	1445	-3152	429	2864	3	-4	8	1	-5				
	12	-738	829	-1000	1397	-3126	358	-1	2	-2	5	2				
2	2	-1062	1030	-353	190	-121	84	2796	-799	408	-252	172	-1	2	-1	2
	4	-3006	-710	1496	-537	304	-201	1950	2371	-725	391	-251	-9	2	-1	0
	6	1271	-2677	-492	1674	-615	358	-1767	1423	2282	-740	414	-3	-7	2	-1
	8	-861	1136	-2532	-375	1773	-662	909	-1926	1104	2242	-756	5	-2	-5	1
	10	662	-772	1069	-2448	-303	1836	-648	937	-1984	900	2218	-2	4	-1	-3
	12	-541	598	-725	1028	-2393	-255	510	-659	932	-2014	757	0	-1	3	1
4	4	16	19	-4	0	1	-1	-2462	1246	-547	327	-220	3220	-1023	558	-360
	6	7	27	9	-5	1	0	-2938	-1540	1427	-587	353	2208	2659	-849	473
	8	-8	-9	14	4	-4	1	1112	-2788	-1152	1567	-631	-1506	1791	2503	-822
	10	8	5	-5	8	2	-2	-727	1092	-2662	-924	1661	789	-1710	1478	2422
	12	-8	-3	3	-3	5	2	552	-721	1057	-2574	-772	-561	851	-1804	1250
6	6							5	8	-2	0	1	-2946	1348	-661	417
	8							1	10	5	-3	0	-2892	-1979	1395	-623
	10							-2	-3	7	3	-2	1028	-2832	-1560	1500
	12							2	1	-2	5	2	-654	1056	-2734	-1298
Coefficients of ip^m_s																
1	1	-245	-126	60	-36	24	-19	111	-57	34	-23	18				
	3	-99	-273	-43	26	-18	16	241	41	-25	18	-16				
	5	193	125	-205	-19	15	-16	-110	195	19	-14	16				
	7	-144	46	124	-156	-14	19	-41	-118	152	14	-19				
	9	116	-40	22	104	-119	-28	35	-21	-101	117	28				
	11	-104	41	-28	26	71	-47	-36	26	-25	-70	46				
3	3	233	63	-74	44	-30	24	-218	-14	9	-7	7	72	-48	34	-30
	5	238	326	-72	-23	17	-17	-342	-134	-12	9	-8	174	32	-24	23
	7	-160	50	229	-82	-15	19	34	-181	-86	-8	7	-32	154	21	-25
	9	124	-41	23	169	-72	-26	-28	16	-109	-58	-7	-32	-51	125	33
	11	-107	40	-27	24	117	-22	25	-15	11	-75	-31	34	-31	-37	51
5	5		-1	0				156	55	-57	40	-34	-157	-5	4	-6
	7		-1	-1				99	216	-24	-25	26	-220	-127	-6	7
	9		1	0				-73	41	175	-38	-31	11	-153	-91	-8
	11		-1	0				63	-39	31	128	-5	-12	11	-114	-50

TABLE XI A (SEE ALSO XI B). $(p_r^n - \Pi_r^n)$ IN TERMS OF p_s^m AND ip_s^m

(Coefficients to 4 places of decimals)

		$n=0$						$n=2$						
m	s	$r=2$	4	6	8	10	12	2	4	6	8	10	12	
0	2	9006	596	412	−322	272	−260	−93	846	307	−233	194	−182	Coefficients of p_s^m
	4	596	7963	985	201	−186	218	−336	−1392	1005	142	−129	148	
	6	412	985	7744	1073	172	−244	462	347	−1579	990	120	−165	
	8	−322	201	1073	7698	1053	317	−358	159	515	−1622	930	213	
	10	272	−186	172	1053	7771	769	296	−144	124	557	−1575	715	
	12	−260	218	−244	317	769	8593	−267	162	−172	219	400	−1043	
2	2	−93	−336	462	−358	296	−267	8898	−478	838	−645	535	−488	
	4	846	−1392	347	159	−144	162	−478	7790	592	367	−321	334	
	6	307	1005	−1579	515	124	−172	838	592	7667	863	267	−326	
	8	−233	142	990	−1622	557	219	−645	367	863	7653	922	380	
	10	194	−129	120	930	−1575	400	535	−321	267	922	7743	678	
	12	−182	148	−165	213	715	−1043	−488	334	−326	380	678	8579	
4	4	−8	4	4	−3	1	−1	816	124	−450	431	−363	341	
	6	−7	−3	8	0	−1	1	611	1062	−678	−28	210	−229	
	8	4	−5	−4	6	−1	0	−433	281	991	−904	103	230	
	10	−3	4	−1	−3	4	−1	345	−231	185	898	−959	44	
	12	3	−3	1	−1	−2	2	−306	220	−199	213	713	−632	
6	6							−1	−3	0	1	−1	0	
	8							0	−2	−3	2	0	0	
	10							0	1	−1	−2	2	0	
	12							0	−1	1	−1	−1	1	
1	1	4274	−1801	1127	−834	664	−549	2374	−1245	797	−600	483	−405	Coefficients of ip_s^m
	3	1146	1481	−647	394	−291	230	−2511	−208	−220	186	−159	139	
	5	−245	840	922	−400	240	−173	1300	−972	−295	−51	67	−66	
	7	87	−247	629	670	−288	165	−896	481	−618	−270	−1	28	
	9	−43	111	−200	500	526	−216	691	−333	293	−450	−238	16	
	11	23	−65	90	−164	410	414	−564	259	−201	205	−353	−205	
3	3	−65	96	−29	2	5	−12	−2315	2257	−1282	913	−718	590	
	5	−125	62	80	−50	21	−8	1326	18	1018	−575	405	−319	
	7	35	−125	47	58	−44	13	−912	284	187	662	−368	253	
	9	−18	56	−87	36	45	−30	708	−254	92	205	489	−260	
	11	10	−37	38	−63	26	20	−583	222	−115	28	193	374	
5	5							−35	−49	37	−9	−2	12	
	7							−10	−70	−10	43	−22	4	
	9							16	18	−60	2	36	−15	
	11							−19	−6	19	−45	4	18	

TABLE XI B (CONTINUING TABLE XI A)

Coefficients of p_s^m

		$n=4$					$n=6$			
m	s	$r=4$	6	8	10	12	6	8	10	12
0	2	−8	−7	4	−3	3				
	4	4	−3	−5	4	−3				
	6	4	8	−4	−1	1				
	8	−3	0	6	−3	−1				
	10	1	−1	−1	4	−2				
	12	−1	1	0	−1	2				
2	2	816	611	−433	345	−306	−1	0	0	0
	4	124	1062	281	−231	220	−3	−2	1	−1
	6	−450	−678	991	185	−199	0	−3	−1	1
	8	431	−28	−904	898	213	1	2	−2	−1
	10	−363	210	103	−959	713	−1	0	2	−1
	12	341	−229	230	44	−632	0	0	0	1
4	4	8016	−791	988	−815	763	1136	626	−478	424
	6	−791	7380	285	512	−538	215	1307	321	−306
	8	988	285	7450	582	531	−483	−480	1172	289
	10	−815	512	582	7605	443	458	−164	−696	948
	12	763	−538	531	443	8501	−451	335	−170	−456
6	6	1136	215	−483	458	−451	8892	−465	517	−461
	8	626	1307	−480	−164	335	−465	8545	48	319
	10	−478	321	1172	−696	−170	517	48	8629	134
	12	424	−306	289	948	−456	−461	319	134	8933

Coefficients of ip_s^m

		$n=4$					$n=6$			
m	s	$r=4$	6	8	10	12	6	8	10	12
1	1	37	17	−19	18	−18				
	3	51	82	−22	11	−4				
	5	−49	−6	72	−28	16				
	7	12	−50	−14	54	−20				
	9	2	27	−38	−16	39				
	11	−10	−9	22	−26	−8				
3	3	2165	−1316	936	−741	621	31	5	−11	15
	5	−2003	161	−381	325	−285	44	61	−16	4
	7	1215	−979	−74	−163	172	−28	16	54	−16
	9	−894	587	−652	−130	−84	4	−34	3	42
	11	713	−427	382	−482	−127	10	12	−26	2
5	5	−2032	1811	−1153	873	−706	1939	−1229	906	−733
	7	1274	−269	933	−586	437	−1623	374	−481	395
	9	−930	435	−7	638	−388	1081	−858	96	−263
	11	752	−371	215	66	471	−848	556	−597	−2

TABLE XII. $(ip_r^n - i\Pi_r^n)$ IN TERMS OF p_s^m AND ip_s^m

(Coefficients to 4 places of decimals)

		$n=1$						$n=3$					$n=5$				
m	s	$r=1$	3	5	7	9	11	3	5	7	9	11	5	7	9	11	
0	2	4274	1146	-245	87	-43	23	-65	-125	35	-18	10					Coefficients of p_s^m
	4	-1801	1481	840	-247	111	-65	96	62	-125	56	-37					
	6	1127	-647	922	629	-200	90	-29	80	47	-87	38					
	8	-834	394	-400	670	500	-164	2	-50	58	36	-63					
	10	664	-291	240	-288	526	410	5	21	-44	45	26					
	12	-549	230	-173	165	-216	414	-12	-8	13	-30	20					
2	2	2374	-2511	1300	-896	691	-564	-2315	1326	-912	708	-583	-35	-10	16	-19	
	4	-1245	-208	-972	481	-333	259	2257	18	284	-254	222	-49	-70	18	-6	
	6	797	-220	-295	-618	293	-201	-1282	1018	187	92	-115	37	-10	-60	19	
	8	-600	186	-51	-270	-450	205	913	-575	662	205	28	-9	43	2	-45	
	10	483	-159	67	-1	-238	-353	-718	405	-368	489	193	-2	-22	36	4	
	12	-405	139	-66	28	16	-205	590	-319	253	-260	374	12	4	-15	18	
4	4	37	51	-49	12	2	-10	2165	-2003	1215	-894	713	-2032	1274	-930	752	
	6	17	82	-6	-50	27	-9	-1316	161	-979	587	-427	1811	-269	435	-371	
	8	-19	-22	72	-14	-38	22	936	-381	-74	-652	382	-1153	933	-7	215	
	10	18	11	-28	54	-16	-26	-741	325	-163	-130	-482	873	-586	638	66	
	12	-18	-4	16	-20	39	-8	621	-285	172	-84	-127	-706	437	-388	471	
6	6							31	44	-28	4	10	1939	-1623	1081	-848	
	8							5	61	16	-34	12	-1229	374	-858	556	
	10							-11	-16	54	3	-26	906	-481	96	-597	
	12							15	4	-16	42	2	-733	395	-263	-2	
1	1	9430	-231	448	-335	271	-241	541	551	-372	289	-249	-1	0	0	-1	Coefficients of ip_s^m
	3	-231	9208	263	242	-203	198	42	868	265	-212	198	-4	-3	2	-1	
	5	448	263	8959	432	156	-180	-404	-820	860	159	-172	1	-3	-2	2	
	7	-335	242	432	8893	468	202	412	73	-1004	794	187	2	3	-2	-1	
	9	271	-203	156	468	8912	339	-336	179	202	-1033	626	-1	0	3	-2	
	11	-241	198	-180	202	339	9349	304	-194	199	139	-689	1	-1	0	1	
3	3	541	42	-404	412	-336	304	8391	-706	960	-772	704	1000	630	-469	406	
	5	551	868	-820	73	179	-194	-706	7569	410	468	-487	181	1196	311	-292	
	7	-372	265	860	-1004	202	199	960	410	7573	688	483	-478	-560	1076	273	
	9	289	-212	159	794	-1033	139	-772	468	688	7685	534	455	-120	-770	864	
	11	-249	198	-172	187	626	-689	704	-487	483	534	8545	-435	314	-112	-519	
5	5	-1	-4	1	2	-1	1	1000	181	-478	455	-435	7760	-861	1042	-975	
	7	0	-3	-3	3	0	-1	630	1196	-560	-120	314	-861	7254	124	723	
	9	0	2	-2	-2	3	0	-469	311	1076	-770	-112	1042	124	7444	178	
	11	-1	-1	2	-1	-2	1	406	-292	273	864	-519	-975	723	178	8405	

Table XIII a and Table XIII b. Expansions for p_r^n (r even) and ip_r^n (r odd)

(Coefficients to 4 places of decimals)

		(*a*) Coefficients of H/h							(*b*) Coefficients of ip_1^1						
n	r	$\beta=2$	4	6	8	10	12	14	2	4	6	8	10	12	14
0	4	−3	−8	−15	−24	−38	−63	−103	−193	−414	−672	−977	−1345	−1805	−2401
	6	−1	−2	−4	−7	−9	−13	−22	54	109	164	219	272	320	359
	8		1	1	2	3	4	4	−24	−48	−73	−100	−128	−159	−191
	10			−1	−1	−2	−2	−2	12	25	38	52	67	83	99
	12			1	1	1	1	1	−7	−15	−23	−32	−41	−51	−62
2	4	4	10	18	29	46	73	109	−132	−283	−455	−654	−884	−1153	−1465
	6				1	1	2	3	38	75	110	143	171	190	201
	8							−1	−17	−34	−53	−72	−94	−117	−144
	10								9	18	28	39	51	64	77
	12								−5	−11	−18	−25	−32	−40	−50
4	4			1	1	1	−2	−6	2	2	−5	−18	−45	−92	−185
	6						2	5	0	−1	−5	−11	−19	−31	−46
	8								−1	−1	−2	−3	−5	−9	−15
	10									1	2	3	4	6	9
	12									−1	−1	−1	−3	−4	−6
6	6											1	0	−2	−5
	8											−1	−1	−2	−4
	10														1
	12														
1	5	−325	−347	−373	−404	−440	−484	−541	32	67	106	150	200	253	308
	7	103	105	108	111	115	119	123	−12	−25	−39	−54	−69	−85	−102
	9	−51	−52	−53	−55	−56	−57	−57	6	13	20	27	35	43	52
	11	28	29	29	30	30	31	32	−4	−8	−12	−17	−22	−27	−33
3	5	2	4	7	10	15	22	30	39	82	131	186	250	327	421
	7	−2	−3	−5	−8	−10	−12	−15	−13	−27	−42	−57	−73	−90	−110
	9			1	1	1	1	1	7	13	20	28	37	47	58
	11								−4	−8	−13	−18	−23	−29	−36
5	5									1	2	5	9	19	40
	7									1	2	4	7	10	11
	9											−1	0	1	1
	11												−1	0	2

TABLE XIII C AND TABLE XIII D. EXPANSIONS FOR p_r^n (r EVEN) AND ip_r^n (r ODD)

(Coefficients to 4 places of decimals)

		(c) Coefficients of p_2^0							(d) Coefficients of p_2^2						
n	r	$\beta = 2$	4	6	8	10	12	14	2	4	6	8	10	12	14
0	4	64	136	220	316	428	555	697	−34	−69	−102	−133	−155	−156	−110
	6	21	43	69	97	130	167	210	22	46	71	96	125	156	188
	8	−9	−18	−26	−35	−43	−49	−54	−10	−21	−32	−43	−55	−68	−82
	10	5	10	15	20	24	28	33	6	11	17	24	30	37	45
	12	−3	−7	−10	−14	−16	−19	−22	−3	−7	−11	−15	−20	−25	−31
2	4	91	197	323	474	661	902	1222	−52	−115	−192	−288	−414	−586	−843
	6	16	33	53	77	105	140	182	41	84	130	178	228	280	332
	8	−6	−12	−18	−22	−26	−29	−29	−18	−37	−57	−77	−98	−120	−144
	10	4	7	10	14	16	18	19	10	20	31	42	54	66	78
	12	−2	−5	−7	−9	−11	−12	−13	−6	−13	−20	−28	−36	−44	−52
4	4	−1	−1	0	1	3	7	16	85	179	282	400	536	700	918
	6		1	3	8	15	27	47	30	61	93	125	156	183	204
	8		1	2	3	6	9	14	−12	−24	−36	−47	−58	−68	−76
	10				−1	−2	−3	−4	6	12	19	25	31	37	43
	12				1	1	2	3	−4	−8	−12	−17	−20	−23	−29
6	6								1	2	6	11	18	28	40
	8									1	3	6	10	15	21
	10										−1	−2	−3	−5	−8
	12											1	2	3	5
1	5	−17	−37	−61	−88	−121	−166	−225	92	194	311	446	603	795	1034
	7	3	7	11	16	21	28	37	−33	−67	−104	−143	−184	−228	−275
	9	−1	−2	−4	−6	−8	−10	−13	16	32	49	67	87	107	129
	11		1	2	2	3	5	6	−9	−18	−28	−38	−49	−61	−73
3	5	−9	−17	−26	−36	−44	−52	−60	92	191	298	414	540	672	806
	7	1	2	4	5	7	9	11	−33	−66	−98	−131	−161	−189	−217
	9		−1	−2	−2	−4	−6	−8	16	32	49	66	83	100	118
	11		1	1	1	2	3	5	−9	−18	−28	−38	−48	−58	−70
5	5			0	−1	−1	−2	−3	−3	−7	−13	−22	−35	−58	−98
	7			−1	−1	−2	−3	−4		3	7	14	24	38	62
	9											−1	−4	−8	−12
	11												1	3	4

TABLE XIII E AND TABLE XIII F. EXPANSIONS FOR p_r^n (r EVEN) AND ip_r^n (r ODD)

(Coefficients to 4 places of decimals)

		(e) Coefficients of ip_3^1							(f) Coefficients of ip_3^3						
n	r	$\beta = 2$	4	6	8	10	12	14	2	4	6	8	10	12	14
0	4	161	354	589	885	1269	1798	2584	6	0	−24	−80	−192	−409	−864
	6	−31	−62	−92	−121	−146	−165	−170		1	5	12	22	33	45
	8	11	23	34	46	60	73	89			−1	−1	−2	−1	−1
	10	−5	−11	−16	−22	−29	−36	−43				−1	−1	−2	−2
	12	3	6	10	13	17	21	26				1	2	3	4
2	4	−25	−62	−116	−198	−325	−538	−922	245	536	888	1327	1893	2653	3775
	6	−10	−17	−21	−21	−16	−2	30	−63	−130	−203	−283	−373	−480	−619
	8	5	10	15	19	23	26	28	26	55	87	123	164	215	275
	10	−3	−6	−8	−11	−14	−16	−17	−14	−28	−45	−63	−85	−113	−144
	12	2	4	5	7	9	11	12	8	17	27	38	52	69	93
4	4	5	7	6	0	−17	−52	−127	238	531	898	1376	2027	2963	4468
	6	4	8	11	14	14	11	0	−64	−132	−204	−281	−367	−469	−590
	8	−1	−2	−3	−5	−7	−11	−17	27	57	90	129	177	241	324
	10			1	2	3	5	9	−14	−30	−47	−67	−93	−127	−172
	12				−1	−2	−3	−5	8	18	29	42	58	81	108
6	6				1	0	0	−2	3	10	22	40	70	114	177
	8				1	1	0	−1		1	2	4	10	23	48
	10							1		−1	−2	−4	−8	−16	−27
	12									1	2	4	6	12	20
1	5	19	43	73	114	170	254	403	−30	−66	−114	−174	−256	−374	−566
	7	9	17	26	34	41	46	46	16	33	53	76	102	135	175
	9	−4	−9	−12	−16	−20	−24	−27	−8	−16	−26	−37	−49	−65	−83
	11	3	6	9	11	14	15	16	5	10	16	23	31	37	41
3	5	61	128	202	287	384	495	632	−53	−122	−212	−335	−508	−765	−1185
	7	10	20	31	43	54	66	77	36	77	125	182	253	347	464
	9	−5	−9	−14	−18	−20	−20	−20	−18	−38	−62	−90	−124	−170	−229
	11	3	6	8	11	13	14	14	11	24	38	55	77	106	144
5	5		0	0	1	4	9	24	65	125	174	202	194	113	−134
	7		1	2	4	8	12	17	24	51	83	121	168	233	316
	9			1	2	3	5	8	−11	−23	−37	−54	−76	−107	−150
	11				−1	−1	−2	−3	7	14	22	32	46	66	88

Table XIV a. Sets of simultaneous equations in ip_1^1, p_2^0, p_2^2, ip_3^1, ip_3^3 and H/h for $\beta = 1$ to 7

β	ip_1^1	p_2^0	p_2^2	ip_3^1	ip_3^3	H/h		
1	−0·5268	0·2132	0·1196	−0·0123	0·0260	0·7665	=	0
	0·0711	−0·8498	−0·0017	0·0191	−0·0009	−0·4574	=	0
	0·0399	−0·0017	−0·8513	−0·0419	−0·0388	0·2633	=	0
	−0·0020	0·0096	−0·0209	−0·9231	0·0004	0·1954	=	0
	0·0043	−0·0004	−0·0194	0·0004	−0·9295	0·0002	=	0
2	−0·0500	0·4255	0·2409	−0·0260	0·0495	0·7655	=	0
	0·1419	−0·6993	−0·0033	0·0383	−0·0015	−0·4573	=	0
	0·0803	−0·0033	−0·7016	−0·0837	−0·0780	0·2623	=	0
	−0·0043	0·0192	−0·0419	−0·8460	0·0006	0·1953	=	0
	0·0082	−0·0008	−0·0390	0·0006	−0·8576	0·0003	=	0
3	0·4307	0·6367	0·3641	−0·0413	0·0705	0·7644	=	0
	0·2122	−0·5486	−0·0051	0·0576	−0·0017	−0·4571	=	0
	0·1214	−0·0051	−0·5509	−0·1255	−0·1177	0·2613	=	0
	−0·0069	0·0288	−0·0628	−0·7685	0·0008	0·1953	=	0
	0·0118	−0·0008	−0·0589	0·0008	−0·7843	0·0005	=	0
4	0·9157	0·8466	0·4893	−0·0585	0·0888	0·7633	=	0
	0·2822	−0·3976	−0·0072	0·0770	−0·0014	−0·4569	=	0
	0·1631	−0·0072	−0·3994	−0·1672	−0·1578	0·2603	=	0
	−0·0097	0·0385	−0·0836	−0·6906	0·0010	0·1952	=	0
	0·0148	−0·0007	−0·0789	0·0010	−0·7093	0·0008	=	0
5	1·4053	1·0551	0·6167	−0·0776	0·1041	0·7621	=	0
	0·3517	−0·2462	−0·0094	0·0965	−0·0007	−0·4567	=	0
	0·2055	−0·0094	−0·2466	−0·2089	−0·1985	0·2592	=	0
	−0·0129	0·0483	−0·1044	−0·6123	0·0010	0·1951	=	0
	0·0174	−0·0004	−0·0993	0·0010	−0·6324	0·0010	=	0
6	1·9000	1·2620	0·7463	−0·0988	0·1159	0·7609	=	0
	0·4207	−0·0945	−0·0118	0·1161	0·0005	−0·4565	=	0
	0·2487	−0·0118	−0·0927	−0·2505	−0·2398	0·2580	=	0
	−0·0165	0·0580	−0·1252	−0·5336	0·0008	0·1950	=	0
	0·0194	0·0003	−0·1198	0·0008	−0·5534	0·0013	=	0
7	2·4004	1·4671	0·8783	−0·1224	0·1242	0·7596	=	0
	0·4890	0·0577	−0·0144	0·1359	0·0025	−0·4563	=	0
	0·2928	−0·0144	0·0624	−0·2920	−0·2816	0·2566	=	0
	−0·0204	0·0679	−0·1460	−0·4543	0·0004	0·1949	=	0
	0·0207	0·0012	−0·1408	0·0004	−0·4717	0·0016	=	0

Table XIV b. Sets of simultaneous equations in ip_1^1, p_2^0, p_2^2, ip_3^1, ip_3^3 and H/h for $\beta = 8$ to 14

β	ip_1^1	p_2^0	p_2^2	ip_3^1	ip_3^3	H/h	
8	2·9070	1·6702	1·0130	−0·1489	0·1280	0·7582	= 0
	0·5567	0·2103	−0·0172	0·1557	0·0052	−0·4560	= 0
	0·3377	−0·0172	0·2189	−0·3333	−0·3241	0·2552	= 0
	−0·0248	0·0778	−0·1667	−0·3743	−0·0005	0·1948	= 0
	0·0213	0·0026	−0·1620	−0·0005	−0·3871	0·0019	= 0
9	3·4205	1·8709	1·1505	−0·1786	0·1276	0·7567	= 0
	0·6236	0·3634	−0·0204	0·1757	0·0087	−0·4558	= 0
	0·3835	−0·0204	0·3769	−0·3744	−0·3674	0·2538	= 0
	−0·0298	0·0879	−0·1872	−0·2935	−0·0016	0·1946	= 0
	0·0213	0·0043	−0·1837	−0·0016	−0·2987	0·0023	= 0
10	3·9420	2·0686	1·2909	−0·2118	0·1215	0·7551	= 0
	0·6895	0·5172	−0·0238	0·1959	0·0135	−0·4554	= 0
	0·4303	−0·0238	0·5364	−0·4153	−0·4115	0·2523	= 0
	−0·0353	0·0980	−0·2076	−0·2118	−0·0036	0·1944	= 0
	0·0203	0·0067	−0·2058	−0·0036	−0·2058	0·0028	= 0
11	4·4725	2·2631	1·4343	−0·2493	0·1096	0·7537	= 0
	0·7544	0·6717	−0·0277	0·2163	0·0193	−0·4551	= 0
	0·4781	−0·0277	0·6976	−0·4559	−0·4566	0·2506	= 0
	−0·0415	0·1081	−0·2279	−0·1289	−0·0063	0·1941	= 0
	0·0183	0·0096	−0·2283	−0·0063	−0·1073	0·0033	= 0
12	5·0132	2·4538	1·5815	−0·2919	0·0907	0·7520	= 0
	0·8179	0·8271	−0·0320	0·2368	0·0266	−0·4547	= 0
	0·5272	−0·0320	0·8609	−0·4959	−0·5028	0·2487	= 0
	−0·0486	0·1184	−0·2479	−0·0444	−0·0104	0·1938	= 0
	0·0152	0·0133	−0·2514	−0·0104	−0·0015	0·0039	= 0
13	5·5663	2·6402	1·7322	−0·3404	0·0648	0·7506	= 0
	0·8801	0·9835	−0·0370	0·2576	0·0359	−0·4542	= 0
	0·5774	−0·0370	1·0263	−0·5351	−0·5499	0·2467	= 0
	−0·0567	0·1288	−0·2675	0·0420	−0·0164	0·1933	= 0
	0·0108	0·0180	−0·2750	−0·0164	0·1141	0·0047	= 0
14	6·1330	2·8209	1·8864	−0·3968	0·0294	0·7492	= 0
	0·9403	1·1411	−0·0428	0·2784	0·0478	−0·4538	= 0
	0·6288	−0·0428	1·1943	−0·5731	−0·5980	0·2445	= 0
	−0·0661	0·1392	−0·2865	0·1312	−0·0251	0·1927	= 0
	0·0047	0·0239	−0·2990	−0·0251	0·2434	0·0055	= 0

Table XV a. Expansions for p_r^n (r even) and ip_r^n (r odd)

(Coefficients to 3 places of decimals)

		Coefficients of H/h				Coefficients of ip_1^1				Coefficients of p_2^0				Coefficients of p_2^2			
n	r	$\beta = 12$	16	20	24	12	16	20	24	12	16	20	24	12	16	20	24
0	8					−15	−21	−27	−35	−6	−8	−11	−14	−6	−9	−12	−14
	10					7	10	12	15	3	4	5	6	3	5	6	7
	12					−4	−6	−8	−10	−2	−3	−4	−5	−2	−3	−4	−5
2	8					−11	−15	−19	−24	−4	−6	−7	−9	−12	−17	−22	−28
	10					5	7	9	11	2	3	4	4	6	9	11	13
	12					−3	−4	−5	−7	−2	−2	−3	−3	−4	−6	−7	−9
4	8							0	0				1	−8	−11	−15	−18
	10							0	0					4	5	7	8
	12							−1	−1					−3	−4	−5	−6
6	8																0
	10																−1
	12																0
1	7	12	13	15	16	−9	−13	−18	−23	2	3	4	5	−23	−33	−44	−57
	9	−6	−6	−6	−7	4	5	6	8	−1	−1	−1	−2	10	14	18	22
	11	3	3	4	4	−2	−3	−4	−5	0	0	1	1	−6	−8	−10	−13
3	7			−1	−1	−10	−14	−19	−24	1	1	1	1	−23	−32	−43	−55
	9			1	1	4	5	6	8			−1	−1	10	14	17	20
	11					−3	−4	−5	−6					−6	−8	−10	−13
5	7						0	0	1							−1	−1
	9						−1	−1	−1							−1	−1
	11															−1	−1

TABLE XV B. EXPANSIONS FOR p_r^n (r EVEN) AND ip_r^n (r ODD)

(Coefficients to 3 places of decimals)

		Coefficients of ip_3^1				Coefficients of ip_3^3				Coefficients of p_4^0				Coefficients of p_4^2			
n	r	$\beta=12$	16	20	24	12	16	20	24	12	16	20	24	12	16	20	24
0	8	8	11	14	18	−1	−2	−3	−4	4	5	6	8	3	4	5	6
	10	−3	−5	−6	−7				1	−2	−3	−4	−4	−2	−2	−3	−3
	12	2	3	4	4					2	3	3	4	1	2	2	3
2	8	3	4	5	7	17	24	32	40	3	4	5	6	7	9	12	15
	10	−2	−2	−3	−4	−9	−12	−15	−18	−1	−2	−2	−3	−4	−5	−7	−8
	12	2	2	3	3	5	7	9	11	1	2	2	2	3	4	5	6
4	8		−1	−1	−1	18	25	33	42					5	8	10	12
	10					−10	−13	−16	−19					−2	−3	−4	−5
	12					5	7	9	12					2	3	3	4
6	8								1								0
	10								1								1
	12								0								0
1	7	6	9	12	16	9	13	17	21	−7	−10	−13	−16	12	18	24	31
	9	−3	−4	−5	−6	−5	−7	−9	−11	2	2	3	3	−5	−7	−9	−11
	11	2	3	4	5	3	4	5	7	−1	−1	−1	−2	3	4	5	6
3	7	7	10	13	16	25	35	47	60	−3	−4	−5	−7	7	10	13	17
	9	−3	−4	−5	−6	−12	−16	−20	−25	1	1	1	1	−3	−5	−6	−7
	11	2	3	3	4	7	10	13	16			−1	−1	2	3	4	5
5	7				−1	17	24	32	41					−2	−2	−3	−4
	9				1	−7	−10	−12	−14						1	1	1
	11				0	4	6	8	9								

TABLE XV c. EXPANSIONS FOR p_r^n (r EVEN) AND ip_r^n (r ODD)

(Coefficients to 3 places of decimals)

		Coefficients of p_4^4				Coefficients of ip_5^1				Coefficients of ip_5^3				Coefficients of ip_5^5			
n	r	$\beta = 12$	16	20	24	12	16	20	24	12	16	20	24	12	16	20	24
0	8		−1	−1	−2	−8	−10	−13	−16	−1	−1						
	10		0	0	1	2	3	3	4	0	0						
	12		0	0	0	−1	−2	−2	−3	0	0						
2	8	8	11	15	19	−1	−1	−1	−1	−12	−16	−20	−25			1	2
	10	−4	−6	−7	−9	0	1	0	0	4	6	8	10			0	−1
	12	3	4	5	7	0	−1	−1	−1	−4	−5	−5	−6			0	0
4	8	19	27	36	46	1	2	2	3	−7	−9	−12	−14	−22	−30	−41	−52
	10	−10	−14	−18	−22		−1	−1	−1	3	5	6	7	10	14	18	23
	12	6	9	12	15					−2	−3	−4	−5	−6	−9	−12	−15
6	8	13	18	25	32					1	2	3	4	−25	−35	−47	−60
	10	−6	−8	−10	−12						−2	−2	−3	11	15	19	24
	12	4	5	6	8								1	−7	−9	−11	−14
1	7	−1	−2	−4	−6	11	15	20	26	2	3	4	5		1	2	3
	9	0	1	1	2	2	3	4	6	3	4	5	6		0	0	−1
	11	0	0	0	0	−2	−3	−4	−5	−2	−3	−3	−4		0	0	0
3	7	30	43	58	74	22	31	41	53	10	14	18	22	−12	−16	−21	−26
	9	−13	−18	−23	−29	3	4	5	7	7	10	13	16	6	9	11	14
	11	7	10	14	17	−1	−2	−3	−3	−5	−7	−8	−10	−5	−7	−9	−11
5	7	33	47	64	83		−1	−2	−3	30	42	56	72	−24	−34	−47	−61
	9	−13	−18	−23	−29		1	1	2	5	7	9	11	16	22	28	35
	11	8	11	14	18				1	−3	−4	−4	−5	−11	−15	−19	−24

TABLE XV D. EXPANSIONS FOR p_r^n (r EVEN) AND ip_r^n (r ODD)

(Coefficients to 3 places of decimals)

		Coefficients of p_6^0				Coefficients of p_6^2				Coefficients of p_6^4				Coefficients of p_6^6			
n	r	$\beta = 12$	16	20	24	12	16	20	24	12	16	20	24	12	16	20	24
0	8	20	28	37	47	9	13	16	20								
	10	2	3	4	6	2	3	4	5								
	12	−2	−2	−3	−3	−1	−2	−2	−2								
2	8	18	25	33	41	16	22	29	37	−1	−2	−2	−3		1	1	2
	10	2	3	4	5	4	6	7	8	3	4	5	6		0	0	0
	12	−1	−2	−2	−2	−2	−3	−4	−5	−2	−3	−4	−4		0	0	0
4	8		−1	−1	−2	19	26	34	42	5	7	9	11	−10	−14	−19	−25
	10				1	3	4	5	6	6	8	11	14	6	8	10	13
	12					−1	−2	−2	−3	−4	−6	−8	−9	−4	−6	−8	−10
6	8							0	−1	25	36	47	60	−10	−15	−20	−26
	10							1	1	4	6	8	10	6	9	12	15
	12							0	0	−2	−3	−4	−5	−4	−6	−8	−10
1	7	17	24	33	44	−16	−24	−33	−44	−1			1				−1
	9	−3	−4	−5	−6	4	6	8	9	0			−1				0
	11	1	1	2	2	−2	−3	−4	−4	0			0				0
3	7	1	1	1	1	5	8	11	16	−25	−35	−47	−61		1	3	5
	9	−1	−1	−1	−1	1	1	2	2	8	11	14	17		0	0	0
	11			1	1	−1	−2	−2	−2	−4	−6	−8	−10		0	0	−1
5	7					0	0	0	1	−6	−9	−11	−14	−42	−60	−81	−106
	9					−1	−1	−1	−1	6	7	9	10	15	22	29	37
	11						1	1	1	−3	−5	−6	−7	−9	−13	−17	−21

TABLE XVI A. SETS OF SIMULTANEOUS EQUATIONS IN ip_1^1, p_2^0, p_2^2, …, p_6^6 AND H/h

(Coefficients to 4 places of decimals)

β	ip_1^1	p_2^0	p_2^2	ip_3^1	ip_3^3	p_4^0	p_4^2	p_4^4	ip_5^1	ip_5^3	ip_5^5	p_6^0	p_6^2	p_6^4	p_6^6	H/h
12	9364	5142	2904	−302	606	−2166	−1520	14	537	675	8	1315	948	40	−1	1527 = 0
16	13177	6863	3901	−414	786	−2890	−2040	4	714	906	16	1734	1260	61	−1	1524 = 0
20	17005	8586	4914	−535	954	−3615	−2569	−15	890	1139	25	2140	1569	90	−2	1521 = 0
24	20850	10315	5947	−664	1107	−4340	−3106	−45	1064	1375	37	2530	1875	127	−5	1517 = 0
12	5142	4814	−106	1371	−83	710	1012	−12	−288	−144	−1	486	361	−8	0	−2744 = 0
16	6863	8421	−138	1824	−112	944	1348	−17	−381	−190	−2	643	478	−11	0	−2743 = 0
20	8586	12030	−170	2277	−143	1178	1683	−22	−473	−235	−3	797	592	−14	−1	−2742 = 0
24	10315	15641	−200	2728	−175	1409	2017	−27	−561	−279	−4	949	702	−17	−2	−2741 = 0
12	2904	−106	4778	−3044	−2873	−398	−621	904	1536	1611	−2	512	1005	777	9	1565 = 0
16	3901	−138	8422	−4075	−3881	−526	−853	1166	2034	2157	17	660	1340	1058	16	1557 = 0
20	4914	−170	12101	−5117	−4918	−652	−1102	1402	2523	2706	47	793	1674	1354	26	1548 = 0
24	5947	−200	15823	−6169	−5989	−774	−1367	1612	2998	3259	90	908	2007	1664	37	1538 = 0
12	−302	1371	−3044	−937	73	1778	−235	78	319	1037	−10	−758	−264	86	1	2350 = 0
16	−414	1824	−4075	2758	108	2371	−307	112	428	1381	−16	−1001	−352	108	1	2353 = 0
20	−535	2277	−5117	6455	151	2963	−373	152	538	1725	−23	−1237	−440	127	3	2355 = 0
24	−664	2728	−6169	10158	201	3555	−435	196	649	2068	−30	−1466	−528	140	5	2358 = 0
12	606	−83	−2873	73	−1796	111	2751	2738	−460	−861	1105	−5	−1514	−1639	−28	9 = 0
16	786	−112	−3881	108	1676	146	3691	3726	−600	−1150	1424	6	−2005	−2215	−71	13 = 0
20	954	−143	−4918	151	5190	180	4644	4762	−729	−1442	1713	28	−2486	−2809	−136	16 = 0
24	1107	−175	−5989	201	8753	212	5611	5852	−846	−1730	1966	59	−2957	−3425	−228	20 = 0
12	−2166	710	−398	1778	111	−10439	−1673	2	1000	70	2	1183	1214	0	0	−5 = 0
16	−2890	944	−526	2371	146	−7249	−2232	1	1328	91	4	1577	1624	1	0	−7 = 0
20	−3615	1178	−652	2963	180	−4058	−2792	−1	1654	112	7	1971	2036	5	0	−9 = 0
24	−4340	1409	−774	3555	212	−865	−3354	−5	1976	131	11	2365	2453	10	0	−11 = 0
12	−1520	1012	−621	−235	2751	−1673	−10628	178	−1157	7	−76	438	710	1259	−5	10 = 0
16	−2040	1348	−853	−307	3691	−2232	−7491	253	−1538	2	−108	595	947	1671	−7	14 = 0
20	−2569	1683	−1102	−373	4644	−2792	−4347	337	−1915	−6	−146	761	1183	2079	−10	19 = 0
24	−3106	2017	−1367	−435	5611	−3354	−1195	431	−2287	−19	−187	934	1419	2481	−12	25 = 0
12	14	−12	904	78	2738	2	178	−10184	−31	−2385	−2599	16	−511	−1009	1240	−1 = 0
16	4	−17	1166	112	3726	1	253	−6805	−26	−3166	−3551	24	−663	−1374	1585	−2 = 0
20	−15	−22	1402	152	4762	−1	337	−3353	−11	−3935	−4560	36	−804	−1758	1886	−3 = 0
24	−45	−27	1612	196	5852	−5	431	180	16	−4690	−5634	48	−928	−2165	2131	−5 = 0

TABLE XVI B. SETS OF SIMULTANEOUS EQUATIONS IN ip_1^1, p_2^0, p_2^2, …, p_6^6 AND H/h

(Coefficients to 4 places of decimals)

β	ip_1^1	p_2^0	p_2^2	ip_3^1	ip_3^3	p_4^0	p_4^2	p_4^4	ip_5^1	ip_5^3	ip_5^5	p_6^0	p_6^2	p_6^4	p_6^6	H/h
12	537	−288	1536	319	−460	1000	−1157	−31	−19214	−968	−11	1104	−357	−31	−1	−914 = 0
16	714	−381	2034	428	−600	1328	−1538	−26	−15601	−1284	−20	1472	−479	−53	−1	−912 = 0
20	890	−473	2523	538	−729	1654	−1915	−11	−11976	−1595	−33	1842	−600	−84	1	−910 = 0
24	1064	−561	2998	649	−846	1976	−2287	16	−8336	−1901	−50	2219	−725	−127	6	−908 = 0
12	675	−144	1611	1037	−861	70	7	−2385	−968	−20841	208	84	1209	189	9	−1 = 0
16	906	−190	2157	1381	−1150	91	2	−3166	−1284	−17750	270	106	1606	248	−13	−1 = 0
20	1139	−235	2706	1725	−1442	112	−6	−3935	−1595	−14634	324	125	1999	306	−54	−2 = 0
24	1375	−279	3259	2068	−1730	131	−19	−4690	−1901	−11489	367	141	2388	359	−117	−2 = 0
12	8	−1	−2	−10	1105	2	−76	−2599	−11	208	−20517	−1	19	2184	2450	0 = 0
16	16	−2	17	−16	1424	4	−108	−3551	−20	270	−17268	−1	13	2917	3334	1 = 0
20	25	−3	47	−23	1713	7	−146	−4560	−33	324	−13958	1	−1	3652	4263	2 = 0
24	37	−4	90	−30	1966	11	−187	−5634	−50	367	−10575	4	−27	4385	5247	3 = 0
12	1315	486	512	−758	−5	1183	438	16	1104	84	−1	−32644	−1876	5	0	10 = 0
16	1734	643	660	−1001	6	1577	595	24	1472	106	−1	−29494	−2493	6	1	15 = 0
20	2140	797	793	−1237	28	1971	761	36	1842	125	1	−26319	−3106	7	1	21 = 0
24	2530	949	908	−1466	59	2365	934	48	2219	141	4	−23121	−3714	8	4	28 = 0
12	948	361	1005	−264	−1514	1214	710	−511	−357	1209	19	−1876	−32736	−809	−10	−11 = 0
16	1260	478	1340	−352	−2005	1624	947	−663	−479	1606	13	−2493	−29613	−1077	−18	−16 = 0
20	1569	592	1674	−440	−2486	2036	1183	−804	−600	1999	−1	−3106	−26468	−1346	−32	−22 = 0
24	1875	702	2007	−528	−2957	2453	1419	−928	−725	2388	−27	−3714	−23300	−1618	−48	−29 = 0
12	40	−8	777	86	−1639	0	1259	−1009	−31	189	2184	5	−809	−33049	275	−1 = 0
16	61	−11	1058	108	−2215	1	1671	−1374	−53	248	2917	6	−1077	−30013	373	−1 = 0
20	90	−14	1354	127	−2809	5	2079	−1758	−84	306	3652	7	−1346	−26945	475	0 = 0
24	127	−17	1664	140	−3425	10	2481	−2165	−127	359	4385	8	−1618	−23844	580	1 = 0
12	−1	0	9	0	−28	0	−5	1240	−1	9	2450	0	−10	275	−31195	0 = 0
16	−1	0	16	1	−71	0	−7	1585	−1	−13	3334	1	−18	373	−27517	0 = 0
20	−2	−1	26	3	−136	0	−10	1886	1	−54	4263	1	−32	475	−23787	0 = 0
24	−5	−2	37	5	−228	0	−12	2131	6	−117	5247	4	−48	580	−19994	0 = 0

TABLE XVII. VALUES OF p_r^n (r EVEN) AND ip_r^n (r ODD) RESULTING FROM THE SOLUTION OF THE EQUATIONS IN TABLE XIV

(Coefficients of H/h)

n	r	$\beta = 1$	2	3	4	5	6	7
1	1	1·3628	−8·945	−0·5244	−0·0601	0·1350	0·870	0·7440
2	0	−0·4205	−2·453	−1·0189	−1·1749	−1·7469	4·630	−0·7359
	2	0·3643	−0·663	0·3306	0·6148	1·4739	−10·985	−1·6306
3	1	0·1960	0·254	0·1936	0·1435	−0·0738	3·423	0·8100
	3	−0·0009	−0·053	−0·0308	−0·0672	−0·2252	2·418	0·5215

n	r	$\beta = 8$	9	10	11	12	13	14
1	1	1·2914	4·367	−2·722	−0·6914	0·453	−3·234	−1·1473
2	0	−1·8628	−6·585	4·032	0·9596	−0·713	4·251	1·2250
	2	−1·3669	−3·139	1·704	0·5017	−0·108	2·544	1·5586
3	1	0·6558	0·238	1·597	1·7035	3·082	−3·773	0·4087
	3	0·6345	2·154	−1·857	−1·1677	−2·210	5·187	1·8360

TABLE XVIII. VALUES OF p_r^n (r EVEN) AND ip_r^n (r ODD) RESULTING FROM THE SOLUTION OF THE EQUATIONS IN TABLE XVI AND EQUATIONS INTERPOLATED FROM THEM

(Coefficients of H/h)

n	r	$\beta = 15$	16	17	18	19	20
1	1	−0·954	−0·134	−0·3334	−0·3162	−0·2909	−0·2762
2	0	0·345	1·752	0·8768	0·6551	0·5215	0·4079
	2	2·994	−4·010	−0·9465	−0·4536	−0·2254	−0·0231
3	1	3·122	−6·055	−1·7456	−0·9765	−0·5980	−0·2757
	3	2·709	−3·197	−0·7211	−0·3524	−0·1982	−0·0729
4	0	0·905	−1·633	−0·5235	−0·3810	−0·3793	−0·5055
	2	0·762	0·104	0·3513	0·4208	0·5235	0·7655
	4	1·815	−2·659	−0·7632	−0·5102	−0·4704	−0·5896
5	1	0·227	−0·802	−0·3485	−0·2906	−0·2916	−0·3459
	3	0·029	−0·237	−0·0742	−0·0214	0·0353	0·1457
	5	−0·141	0·298	0·1168	0·1045	0·1280	0·2118
6	0	−0·003	0·010	−0·0036	−0·0112	−0·0210	−0·0409
	2	−0·078	0·093	0·0267	0·0202	0·0222	0·0321
	4	−0·124	0·223	0·0844	0·0736	0·0862	0·1310
	6	0·075	−0·108	−0·0293	−0·0171	−0·0118	−0·0065

TABLE XIX A. VALUES OF $p_r^n \pi_r^n$ (r EVEN) AND $ip_r^n \pi_r^n$ (r ODD)

(Coefficients of H/h)

n	r	$\beta = 1$	2	3	4	5	6	7	8	9	10
0	2	−0·0969	−0·5649	−0·2347	−0·2706	−0·4024	1·0664	−0·1695	−0·4291	−1·5169	0·9286
	4	−0·0017	0·0206	0·0010	−0·0017	−0·0069	0·0441	−0·0008	−0·0147	−0·0891	0·0942
	6	0·0003	−0·0049	−0·0007	−0·0003	0·0001	−0·0054	−0·0012	−0·0009	−0·0001	−0·0025
	8	−0·0001	0·0016	0·0002	0·0001	−0·0001	0·0019	0·0003	0·0002	−0·0005	0·0012
	10		−0·0007	−0·0001			−0·0008	−0·0002	−0·0001	0·0001	−0·0004
	12		0·0003				0·0004	0·0001	0·0001		0·0002
2	2	0·1187	−0·2160	0·1077	0·2003	0·4801	−3·5782	−0·5311	−0·4452	−1·0225	0·5552
	4	−0·0021	0·0174	−0·0014	−0·0057	−0·0154	0·0888	0·0028	−0·0106	−0·0455	0·0067
	6	0·0004	−0·0050	−0·0004	0·0002	0·0016	−0·0203	−0·0043	−0·0049	−0·0153	0·0125
	8	−0·0001	0·0017	0·0001	−0·0001	−0·0006	0·0071	0·0014	0·0013	0·0036	−0·0027
	10		−0·0007	−0·0001		0·0003	−0·0031	−0·0006	−0·0006	−0·0017	0·0012
	12		0·0004			−0·0001	0·0016	0·0003	0·0003	0·0009	−0·0006
4	4	0·0003	−0·0015	0·0006	0·0014	0·0032	−0·0162	0·0005	0·0054	0·0358	−0·0489
	6	0·0001	−0·0002	0·0002	0·0006	0·0018	−0·0181	−0·0037	−0·0045	−0·0156	0·0133
	8		0·0002	−0·0001	−0·0002	−0·0006	0·0057	0·0011	0·0013	0·0042	−0·0038
	10				0·0001	0·0003	−0·0024	−0·0005	−0·0005	−0·0018	0·0016
	12					−0·0001	0·0013	0·0003	0·0003	0·0010	−0·0008
6	6						−0·0002		0·0001	0·0009	−0·0012
	8						−0·0003			−0·0001	0·0000
	10									0·0000	0·0001
	12									0·0001	−0·0001
1	1	0·7689	−5·0467	−0·2959	−0·0339	0·0762	0·4906	0·4198	0·7286	2·4647	−1·5360
	3	0·0451	0·0585	0·0446	0·0331	−0·0170	0·7885	0·1866	0·1511	0·0548	0·3678
	5	−0·0040	−0·0091	−0·0041	−0·0026	0·0018	−0·0583	−0·0124	−0·0101	−0·0151	0·0044
	7	0·0009	0·0024	0·0010	0·0006	−0·0005	0·0158	0·0035	0·0029	0·0047	−0·0005
	9	−0·0004	−0·0010	−0·0004	−0·0003	0·0002	−0·0059	−0·0013	−0·0011	−0·0018	0·0002
	11	0·0002	0·0005	0·0002	0·0001	−0·0001	0·0028	0·0006	0·0005	0·0009	−0·0001
3	3	−0·0002	−0·0121	−0·0071	−0·0155	−0·0519	0·5569	0·1201	0·1461	0·4961	−0·4277
	5	0·0007	−0·0053	0·0008	0·0024	0·0064	−0·0451	−0·0054	−0·0040	−0·0158	0·0238
	7	−0·0002	0·0014	−0·0001	−0·0005	−0·0017	0·0156	0·0027	0·0025	0·0066	−0·0047
	9	0·0001	−0·0006		0·0002	0·0006	−0·0061	−0·0011	−0·0010	−0·0026	0·0019
	11		0·0003		−0·0001	−0·0003	0·0029	0·0005	0·0005	0·0013	−0·0009
5	5			−0·0001	−0·0002	−0·0007	0·0083	0·0019	0·0024	0·0082	−0·0065
	7					−0·0001	0·0014	0·0004	0·0007	0·0031	−0·0030
	9					0·0001	−0·0007	−0·0002	−0·0003	−0·0011	0·0012
	11						0·0004	0·0001	0·0001	0·0006	−0·0006

TABLE XIX B. VALUES OF $p_r^n \pi_r^n$ (r EVEN) AND $ip_r^n \pi_r^n$ (r ODD)

(Coefficients of H/h)

n	r	$\beta = 11$	12	13	14	15	16	17	18	19	20
0	2	0·2210	−0·1643	0·9790	0·2822	0·0795	0·4035	0·2020	0·1509	0·1201	0·0940
	4	0·0545	0·0654	−0·0285	0·0354	0·1142	−0·2060	−0·0660	−0·0481	−0·0478	−0·0638
	6	−0·0026	−0·0051	0·0083	0·0011	−0·0003	0·0009	−0·0003	−0·0010	−0·0018	−0·0036
	8	0·0010	0·0013	−0·0010	0·0004	0·0014	−0·0021	−0·0004	−0·0001	0·0001	0·0003
	10	−0·0004	−0·0005	0·0004	−0·0002	−0·0005	0·0008	0·0002			−0·0001
	12	0·0002	0·0002	−0·0002	0·0001	0·0003	−0·0004	−0·0001			0·0001
2	2	0·1634	−0·0352	0·8288	0·5077	0·9753	−1·3062	−0·3083	−0·1478	−0·0734	−0·0075
	4	−0·0370	−0·1526	0·4638	0·1521	0·1359	0·0186	0·0627	0·0751	0·0934	0·1366
	6	0·0074	0·0125	−0·0250	−0·0075	−0·0096	0·0115	0·0033	0·0025	0·0027	0·0040
	8	−0·0017	−0·0039	0·0106	0·0039	0·0061	−0·0070	−0·0016	−0·0007	−0·0005	−0·0005
	10	0·0007	0·0016	−0·0043	−0·0016	−0·0025	0·0029	0·0006	0·0003	0·0002	0·0002
	12	−0·0004	−0·0008	0·0022	0·0009	0·0013	−0·0023	−0·0003	−0·0002	−0·0001	−0·0001
4	4	−0·0455	−0·1219	0·3843	0·1750	0·3238	−0·4744	−0·1362	−0·0910	−0·0839	−0·1052
	6	0·0078	0·0126	−0·0244	−0·0080	−0·0153	0·0275	0·0104	0·0091	0·0106	0·0161
	8	−0·0026	−0·0054	0·0132	0·0047	0·0076	−0·0099	−0·0026	−0·0017	−0·0016	−0·0023
	10	0·0011	0·0023	−0·0055	−0·0020	−0·0032	0·0042	0·0011	0·0008	0·0008	0·0011
	12	−0·0006	−0·0012	0·0030	0·0010	0·0017	−0·0023	−0·0006	−0·0004	−0·0004	−0·0006
6	6	−0·0011	−0·0032	0·0104	0·0048	0·0092	−0·0133	−0·0036	−0·0021	−0·0015	−0·0008
	8	−0·0001	−0·0005	0·0022	0·0012	0·0029	−0·0049	−0·0016	−0·0012	−0·0012	−0·0017
	10	0·0001	0·0003	−0·0010	−0·0005	−0·0012	0·0020	0·0007	0·0005	0·0006	0·0008
	12	−0·0001	−0·0002	0·0006	0·0003	0·0006	−0·0011	−0·0003	−0·0003	−0·0003	−0·0004
1	1	−0·3901	0·2554	−1·8246	−0·6473	−0·5383	−0·0756	−0·1881	−0·1784	−0·1641	−0·1558
	3	0·3924	0·7099	−0·8690	0·0941	0·7190	−1·3946	−0·4021	−0·2249	−0·1377	−0·0635
	5	0·0045	0·0185	−0·0512	−0·0063	0·0331	−0·1168	−0·0508	−0·0423	−0·0425	−0·0504
	7	0·0003	−0·0007	0·0058	0·0021	0·0002	0·0064	0·0036	0·0033	0·0033	0·0038
	9	−0·0001	0·0002	−0·0020	−0·0008	−0·0002	−0·0024	−0·0013	−0·0012	−0·0012	−0·0013
	11	0·0001	−0·0001	0·0009	0·0003	0·0003	0·0007	0·0005	0·0005	0·0005	0·0006
3	3	−0·2690	−0·5091	1·1946	0·4229	0·6239	−0·7364	−0·1661	−0·0812	−0·0457	−0·0168
	5	0·0226	0·0488	−0·0952	−0·0173	0·0042	−0·0345	−0·0108	−0·0031	0·0051	0·0212
	7	−0·0030	−0·0064	0·0176	0·0071	0·0132	−0·0193	−0·0057	−0·0040	−0·0037	−0·0047
	9	0·0012	0·0028	−0·0073	−0·0027	−0·0043	0·0053	0·0013	0·0008	0·0008	0·0012
	11	−0·0006	−0·0014	0·0038	0·0014	0·0023	−0·0028	−0·0007	−0·0004	−0·0004	−0·0006
5	5	−0·0032	−0·0030	−0·0035	−0·0064	−0·0206	0·0434	0·0170	0·0152	0·0186	0·0309
	7	−0·0022	−0·0051	0·0153	0·0071	0·0145	−0·0226	−0·0068	−0·0046	−0·0041	−0·0047
	9	0·0009	0·0021	−0·0060	−0·0025	−0·0046	0·0067	0·0020	0·0014	0·0014	0·0020
	11	−0·0005	−0·0011	0·0029	0·0011	0·0023	−0·0034	−0·0010	−0·0007	−0·0007	−0·0010

Table XX a. Values of $p^n_{-r}\,\pi^n_r$ (r odd) and $ip^n_{-r}\,\pi^n_r$ (r even)

(Coefficients of H/h)

n	r	$\beta=1$	2	3	4	5	6	7	8	9	10
2	3	−0·0866	0·4360	0·0071	−0·0182	−0·0315	−0·0332	−0·0545	−0·0802	−0·1927	0·0318
	5	0·0059	−0·0834	−0·0102	−0·0046	0·0044	−0·1152	−0·0242	−0·0185	−0·0084	−0·0362
	7	−0·0022	0·0307	0·0035	0·0014	−0·0019	0·0381	0·0073	0·0052	0·0032	0·0062
	9	0·0011	−0·0146	−0·0017	−0·0007	0·0009	−0·0185	−0·0036	−0·0026	−0·0022	−0·0024
	11	−0·0006	0·0083	0·0010	0·0004	−0·0006	0·0111	0·0022	0·0016	0·0017	0·0010
4	5	0·0003	0·0032	0·0016	0·0030	0·0095	−0·1019	−0·0223	−0·0273	−0·0914	0·0757
	7	−0·0004	0·0005	−0·0006	−0·0013	−0·0038	0·0333	0·0061	0·0068	0·0231	−0·0217
	9	0·0002	−0·0005	0·0003	0·0006	0·0018	−0·0166	−0·0031	−0·0034	−0·0110	0·0097
	11	−0·0001	0·0003	−0·0002	−0·0004	−0·0011	0·0098	0·0018	0·0020	0·0063	−0·0055
6	7				0·0001	0·0002	−0·0028	−0·0007	−0·0009	−0·0031	0·0026
	9					−0·0001	0·0005	0·0001	0·0001	0·0001	
	11						−0·0001			0·0002	−0·0002
1	2	−0·0465	−0·0891	−0·0703	−0·0878	−0·1422	0·5482	0·0024	−0·0575	−0·2121	0·1045
	4	−0·0163	−0·0926	−0·0373	−0·0404	−0·0501	−0·0025	−0·0647	−0·1105	−0·3575	0·2144
	6	0·0025	0·0224	0·0068	0·0063	0·0056	0·0231	0·0127	0·0167	0·0432	−0·0186
	8	−0·0009	−0·0095	−0·0028	−0·0026	−0·0023	−0·0110	−0·0059	−0·0081	−0·0228	0·0117
	10	0·0005	0·0050	0·0014	0·0013	0·0011	0·0062	0·0031	0·0042	0·0118	−0·0060
	12	−0·0003	−0·0029	−0·0008	−0·0007	−0·0006	−0·0039	−0·0019	−0·0025	−0·0071	0·0036
3	4	0·0213	−0·0423	0·0165	0·0301	0·0696	−0·5025	−0·0738	−0·0634	−0·1557	0·0930
	6	−0·0048	0·0140	−0·0032	−0·0068	−0·0165	0·1159	0·0150	0·0101	0·0197	−0·0145
	8	0·0019	−0·0065	0·0011	0·0026	0·0065	−0·0487	−0·0070	−0·0056	−0·0129	0·0090
	10	−0·0010	0·0035	−0·0005	−0·0013	−0·0033	0·0253	0·0037	0·0030	0·0069	−0·0047
	12	0·0006	−0·0022	0·0003	0·0007	0·0020	−0·0152	−0·0023	−0·0018	−0·0043	0·0029
5	6	0·0001	−0·0005	0·0002	0·0005	0·0012	−0·0076	−0·0007	0·0003	0·0047	−0·0077
	8		0·0001			0·0001	−0·0022	−0·0007	−0·0011	−0·0047	0·0049
	10					−0·0001	0·0017	0·0004	0·0006	0·0026	−0·0027
	12					0·0001	−0·0016	−0·0004	−0·0005	−0·0018	0·0018

TABLE XX B. VALUES OF $p^n_{-r}\pi^n_r$ (r ODD) AND $ip^n_{-r}\pi^n_r$ (r EVEN)

(Coefficients of H/h)

n	r	$\beta=11$	12	13	14	15	16	17	18	19	20
2	3	−0·0662	−0·2173	0·4944	−0·1115	0·0412	0·0841	0·0419	0·0285	0·0201	0·0138
	5	−0·0358	−0·0519	0·0060	−0·0501	−0·1459	0·2396	0·0652	0·0359	0·0225	0·0119
	7	0·0048	0·0019	0·0230	0·0165	0·0306	−0·0356	−0·0063	−0·0014	0·0010	0·0033
	9	−0·0020	−0·0005	−0·0115	−0·0081	−0·0156	0·0194	0·0040	0·0015	0·0003	−0·0006
	11	0·0009	−0·0001	0·0073	0·0048	0·0091	−0·0110	−0·0022	−0·0008	−0·0002	0·0003
4	5	0·0451	0·0821	−0·1935	−0·0720	−0·1138	0·1452	0·0354	0·0188	0·0119	0·0068
	7	−0·0149	−0·0290	0·0637	0·0195	0·0235	−0·0231	−0·0050	−0·0032	−0·0036	−0·0063
	9	0·0064	0·0124	−0·0287	−0·0097	−0·0138	0·0165	0·0041	0·0026	0·0024	0·0033
	11	−0·0036	−0·0072	0·0168	0·0057	0·0083	−0·0100	−0·0025	−0·0016	−0·0015	−0·0020
6	7	0·0015	0·0022	−0·0031	−0·0002	0·0015	−0·0050	−0·0024	−0·0025	−0·0034	−0·0061
	9	0·0001	0·0005	−0·0024	−0·0015	−0·0037	0·0065	0·0022	0·0017	0·0018	0·0026
	11	−0·0002	−0·0006	0·0023	0·0011	0·0024	−0·0040	−0·0013	−0·0010	−0·0011	−0·0016
1	2	0·0068	−0·0369	−0·0014	−0·0784	−0·2443	0·4104	0·1099	0·0568	0·0302	0·0059
	4	0·0533	−0·0316	0·2307	0·0824	0·0747	−0·0137	0·0124	0·0118	0·0082	0·0014
	6	−0·0011	0·0076	−0·0121	0·0038	0·0196	−0·0378	−0·0113	−0·0070	−0·0056	−0·0059
	8	0·0020	−0·0027	0·0098	0·0014	−0·0028	0·0091	0·0032	0·0020	0·0014	0·0011
	10	−0·0011	0·0012	−0·0045	−0·0005	0·0018	−0·0051	−0·0017	−0·0010	−0·0007	−0·0006
	12	0·0007	−0·0006	0·0024	0·0002	−0·0012	0·0031	0·0010	0·0006	0·0004	0·0003
3	4	0·0314	0·0042	0·1068	0·0649	0·1094	−0·1110	−0·0126	0·0062	0·0196	0·0411
	6	−0·0119	−0·0295	0·0742	0·0219	0·0179	0·0028	0·0075	0·0082	0·0094	0·0120
	8	0·0055	0·0106	−0·0227	−0·0062	−0·0053	0·0013	0·0007	−0·0008	−0·0006	−0·0003
	10	−0·0028	−0·0055	0·0123	0·0036	0·0037	−0·0021		0·0002	0·0002	−0·0001
	12	0·0017	0·0034	−0·0079	−0·0024	−0·0028	0·0021	0·0002			0·0003
5	6	−0·0079	−0·0226	0·0748	0·0350	0·0646	−0·0937	−0·0263	−0·0170	−0·0151	−0·0182
	8	0·0037	0·0082	−0·0229	−0·0099	−0·0184	0·0281	0·0086	0·0061	0·0061	0·0082
	10	−0·0020	−0·0046	0·0128	0·0054	0·0098	−0·0143	−0·0042	−0·0029	−0·0028	−0·0037
	12	0·0013	0·0030	−0·0082	−0·0034	−0·0060	0·0088	0·0025	0·0018	0·0017	0·0023

Table XXI a. Values of ϕ

	Coefficients of $H/h\,.\cos s\theta\cos n\chi\,.\,e^{i\sigma t}$					Coefficients of $-H/h\,.\,i\sin s\theta\cos n\chi\,.\,e^{i\sigma t}$			
β	s	$n=0$	2	4	6	s	$n=1$	3	5
1	0	−0·0388	0·0569	0·0002		1	0·6747		
	2	−0·1159	−0·0582	−0·0002		3	0·0444	0·0005	
	4	−0·0018	0·0016			5	−0·0039	−0·0004	
	6	0·0003	−0·0003			7	0·0008	0·0002	
	8	−0·0001				9	−0·0003	−0·0001	
	10					11	0·0002		
	12								
2	0	−0·2181	−0·1001	−0·0006		1	−4·3594	−0·0109	
	2	−0·6582	0·1103	0·0009		3	0·0564	0·0001	
	4	0·0214	−0·0132	−0·0002		5	−0·0087	0·0031	
	6	−0·0049	0·0042	−0·0002		7	0·0023	−0·0010	
	8	0·0016	−0·0015	0·0001		9	−0·0009	0·0006	
	10	−0·0007	0·0006			11	0·0005	−0·0003	
	12	0·0003	−0·0003						
3	0	−0·0926	0·0516	0·0003		1	−0·2476	−0·0054	
	2	−0·2780	−0·0530	−0·0003		3	0·0439	0·0024	
	4	0·0008	0·0010	−0·0001		5	−0·0040	−0·0005	
	6	−0·0007	0·0003	0·0001		7	0·0010	0·0001	
	8	0·0002	−0·0001			9	−0·0003		
	10	−0·0001	0·0002			11	0·0002		
	12								
4	0	−0·1075	0·0952	0·0007		1	−0·0229	−0·0114	
	2	−0·3221	−0·0993	−0·0007		3	0·0327	0·0055	
	4	−0·0022	0·0042	−0·0002		5	−0·0026	−0·0013	
	6	−0·0003	−0·0002	0·0003		7	0·0005	0·0003	
	8	0·0001	0·0001	−0·0001		9	−0·0003	−0·0002	
	10					11	0·0001	0·0001	
	12								
5	0	−0·1611	0·2280	0·0017		1	0·0627	−0·0389	−0·0005
	2	−0·4818	−0·2384	−0·0016		3	−0·0167	0·0175	0·0004
	4	−0·0080	0·0114	−0·0007		5	0·0017	−0·0038	
	6	0·0001	−0·0013	0·0009		7	−0·0005	0·0011	−0·0001
	8	−0·0001	0·0005	−0·0004		9	0·0002	−0·0005	
	10		−0·0003	0·0002		11	−0·0001	0·0002	
	12		0·0001	−0·0001					

TABLE XXI B. VALUES OF ϕ

		Coefficients of $H/h\,.\cos s\theta\cos n\chi\,.\,e^{i\sigma t}$					Coefficients of $-H/h\,.\,i\sin s\theta\cos n\chi\,.\,e^{i\sigma t}$		
β	s	$n=0$	2	4	6	s	$n=1$	3	5
6	0	0·4337	−1·7083	−0·0107	−0·0002	1	0·5797	0·4247	0·0064
	2	1·2917	1·7630	0·0076	0·0001	3	0·7806	−0·1710	−0·0023
	4	0·0485	−0·0667	0·0097	0·0001	5	−0·0560	0·0283	−0·0002
	6	−0·0053	0·0169	−0·0088		7	0·0151	−0·0107	0·0007
	8	0·0019	−0·0060	0·0033		9	−0·0051	0·0051	−0·0005
	10	−0·0007	0·0027	−0·0019		11	0·0031	−0·0022	0·0001
	12	0·0005	−0·0016	0·0008					
7	0	−0·0675	−0·2571	−0·0006		1	0·4003	0·0928	0·0015
	2	−0·2021	0·2569	−0·0006		3	0·1852	−0·0342	−0·0005
	4	−0·0016	−0·0025	0·0025		5	−0·0119	0·0037	−0·0001
	6	−0·0013	0·0036	−0·0018		7	0·0033	−0·0019	0·0002
	8	0·0003	−0·0012	0·0006		9	−0·0011	0·0010	−0·0001
	10	−0·0002	0·0005	−0·0004		11	0·0007	−0·0004	
	12	0·0002	−0·0002	0·0003					
8	0	−0·1742	−0·2199	0·0012		1	0·6607	0·1137	0·0019
	2	−0·5190	0·2094	−0·0033		3	0·1499	−0·0400	−0·0006
	4	−0·0175	0·0073	0·0037		5	−0·0096	0·0030	−0·0003
	6	−0·0009	0·0041	−0·0021		7	0·0028	−0·0018	0·0003
	8	0·0002	−0·0011	0·0007		9	−0·0010	0·0008	−0·0002
	10	−0·0001	0·0005	−0·0004		11	0·0005	−0·0003	0·0001
	12	0·0001	−0·0003	0·0002					
9	0	−0·6263	−0·5126	0·0013	0·0003	1	2·1443	0·3850	0·0069
	2	−1·8581	0·4704	−0·0212	−0·0005	3	0·0510	−0·1381	−0·0018
	4	−0·1036	0·0318	0·0154	0·0002	5	−0·0143	0·0104	−0·0013
	6	−0·0004	0·0128	−0·0072	−0·0001	7	0·0045	−0·0045	0·0013
	8	−0·0005	−0·0030	0·0024	−0·0001	9	−0·0015	0·0022	−0·0007
	10	0·0001	0·0015	−0·0015	0·0001	11	0·0010	−0·0010	0·0002
	12		−0·0009	0·0008	0·0001				
10	0	0·3948	0·2736	−0·0173	−0·0005	1	−1·2553	−0·3286	−0·0056
	2	1·1628	−0·2614	0·0282	0·0007	3	0·3739	0·1268	0·0013
	4	0·1082	−0·0036	−0·0156	−0·0003	5	0·0045	−0·0135	0·0013
	6	−0·0023	−0·0105	0·0062	0·0001	7	−0·0005	0·0032	−0·0014
	8	0·0012	0·0023	−0·0022	0·0001	9	0·0002	−0·0016	0·0008
	10	−0·0003	−0·0010	0·0013	−0·0001	11	−0·0001	0·0007	−0·0002
	12	0·0002	0·0006	−0·0006					

TABLE XXI C. VALUES OF ϕ

	Coefficients of $H/h\,.\,\cos s\theta\cos n\chi\,.\,e^{i\sigma t}$					Coefficients of $-H/h\,.\,i\sin s\theta\cos n\chi\,.\,e^{i\sigma t}$			
β	s	$n=0$	2	4	6	s	$n=1$	3	5
11	0	0·1031	0·0690	−0·0172	−0·0004	1	−0·2579	−0·2040	−0·0029
	2	0·2972	−0·0920	0·0258	0·0006	3	0·3990	0·0850	0·0004
	4	0·0620	0·0279	−0·0114	−0·0002	5	0·0049	−0·0123	0·0011
	6	−0·0025	−0·0062	0·0039		7	0·0003	0·0020	−0·0010
	8	0·0010	0·0014	−0·0015	0·0001	9	−0·0001	−0·0010	0·0006
	10	−0·0003	−0·0006	0·0009	−0·0001	11	0·0001	0·0005	−0·0002
	12	0·0002	0·0005	−0·0005					
12	0	−0·0465	−0·0625	−0·0482	−0·0013	1	0·3668	−0·3845	−0·0037
	2	−0·1537	−0·0428	0·0683	0·0018	3	0·7252	0·1648	−0·0008
	4	0·0732	0·1130	−0·0249	−0·0005	5	0·0192	−0·0262	0·0028
	6	−0·0052	−0·0104	0·0069	−0·0001	7	−0·0007	0·0044	−0·0023
	8	0·0011	0·0033	−0·0032	0·0003	9	0·0002	−0·0023	0·0014
	10	−0·0004	−0·0014	0·0018	−0·0002	11	−0·0001	0·0010	−0·0004
	12	0·0003	0·0008	−0·0007					
13	0	0·3804	0·5426	0·1555	0·0044	1	−1·7610	0·9089	0·0021
	2	1·1460	−0·2142	−0·2138	−0·0059	3	−0·8969	−0·3722	0·0068
	4	−0·0283	−0·3418	0·0682	0·0013	5	−0·0519	0·0536	−0·0095
	6	0·0090	0·0208	−0·0149	0·0008	7	0·0056	−0·0119	0·0066
	8	−0·0010	−0·0090	0·0076	−0·0010	9	−0·0017	0·0061	−0·0037
	10	0·0003	0·0037	−0·0044	0·0006	11	0·0009	−0·0029	0·0011
	12	−0·0001	−0·0021	0·0018	−0·0002				
14	0	0·1224	0·2924	0·0714	0·0021	1	−0·5420	0·3271	−0·0025
	2	0·3588	−0·1840	−0·0972	−0·0027	3	0·0934	−0·1197	0·0051
	4	0·0419	−0·1119	0·0291	0·0005	5	−0·0059	0·0114	−0·0048
	6	0·0015	0·0062	−0·0051	0·0005	7	0·0020	−0·0048	0·0029
	8	0·0004	−0·0033	0·0028	−0·0005	9	−0·0007	0·0023	−0·0015
	10	−0·0002	0·0014	−0·0016	0·0003	11	0·0002	−0·0011	0·0004
	12	0·0002	−0·0008	0·0006	−0·0002				
15	0	0·0657	0·5135	0·1318	0·0041	1	−0·3172	0·4930	−0·0105
	2	0·1704	−0·4174	−0·1802	−0·0053	3	0·7397	−0·1544	0·0130
	4	0·1329	−0·0998	0·0544	0·0006	5	0·0350	0·0037	−0·0103
	6	0·0004	0·0080	−0·0090	0·0012	7	0·0002	−0·0087	0·0056
	8	0·0014	−0·0052	−0·0044	−0·0012	9	−0·0001	0·0036	−0·0029
	10	−0·0004	0·0021	−0·0026	0·0006	11	0·0003	−0·0018	0·0009
	12	0·0003	−0·0012	0·0012					

TABLE XXI D. VALUES OF ϕ

	Coefficients of $H/h \,.\, \cos s\theta \cos n\chi \,.\, e^{i\sigma t}$					Coefficients of $-H/h \,.\, i \sin s\theta \cos n\chi \,.\, e^{i\sigma t}$			
β	s	$n = 0$	2	4	6	s	$n = 1$	3	5
16	0	0·0979	−0·6250	−0·1915	−0·0061	1	−0·3593	−0·5921	0·0245
	2	0·3416	0·6426	0·2649	0·0076	3	−1·4522	0·1595	−0·0247
	4	−0·2393	−0·0137	−0·0837	−0·0005	5	−0·1209	0·0082	0·0168
	6		−0·0097	0·0140	−0·0021	7	0·0060	0·0127	−0·0084
	8	−0·0021	0·0058	−0·0057	0·0019	9	−0·0023	−0·0045	0·0043
	10	0·0007	−0·0023	0·0034	−0·0011	11	0·0008	0·0022	−0·0013
	12	−0·0005	0·0023	−0·0014	0·0003				
17	0	0·0600	−0·1290	−0·0542	−0·0017	1	−0·2492	−0·1348	0·0102
	2	0·1955	0·1764	0·0765	0·0021	3	−0·4244	0·0332	−0·0090
	4	−0·0769	−0·0459	−0·0259		5	−0·0523	0·0026	0·0054
	6	−0·0005	−0·0027	0·0047	−0·0008	7	0·0034	0·0038	−0·0025
	8	−0·0004	0·0014	−0·0015	0·0007	9	−0·0012	−0·0011	0·0013
	10	0·0002	−0·0005	0·0009	−0·0003	11	0·0006	0·0005	−0·0004
	12	−0·0002	0·0003	−0·0005					
18	0	0·0450	−0·0474	−0·0356	−0·0011	1	−0·2041	−0·0653	0·0095
	2	0·1465	0·1039	0·0514	0·0012	3	−0·2420	0·0172	−0·0073
	4	−0·0565	−0·0549	−0·0189	0·0001	5	−0·0434	−0·0003	0·0039
	6	−0·0012	−0·0021	0·0037	−0·0006	7	0·0031	0·0026	−0·0017
	8	−0·0001	0·0006	−0·0010	0·0005	9	−0·0011	−0·0007	0·0009
	10		−0·0002	0·0006	−0·0003	11	0·0005	0·0003	−0·0003
	12		0·0001	−0·0002	0·0002				
19	0	0·0328	−0·0056	−0·0322	−0·0008	1	−0·1741	−0·0349	0·0123
	2	0·1098	0·0757	0·0477	0·0009	3	−0·1538	0·0138	−0·0083
	4	−0·0565	−0·0682	−0·0190	0·0002	5	−0·0436	−0·0042	0·0038
	6	−0·0020	−0·0023	0·0041	−0·0006	7	0·0031	0·0025	−0·0016
	8	0·0001	0·0004	−0·0010	0·0005	9	−0·0011	−0·0007	0·0009
	10		−0·0002	0·0006	−0·0003	11	0·0006	0·0003	−0·0003
	12		0·0002	−0·0002	0·0001				
20	0	0·0173	0·0403	−0·0397	−0·0006	1	−0·1526	−0·0070	0·0211
	2	0·0674	0·0625	0·0601	0·0005	3	−0·0814	0·0171	−0·0129
	4	−0·0761	−0·0997	−0·0258	0·0006	5	−0·0518	−0·0125	0·0050
	6	−0·0040	−0·0034	0·0062	−0·0009	7	0·0037	0·0031	−0·0021
	8	0·0003	0·0004	−0·0014	0·0007	9	−0·0011	−0·0010	0·0013
	10	−0·0001	−0·0002	0·0009	−0·0004	11	0·0006	0·0005	−0·0004
	12	0·0002	0·0001	−0·0003	0·0001				

TABLE XXII A. VALUES OF ψ

	Coefficients of $H/h\,.\cos s\theta \sin n\chi\,.\,e^{i\sigma t}$				Coefficients of $-H/h\,.\,i \sin s\theta \sin n\chi\,.\,e^{i\sigma t}$			
β	s	$n=2$	4	6	s	$n=1$	3	5
1	1	−0·0531			2	−0·0495	0·0150	
	3	0·0564	−0·0002		4	−0·0161	−0·0106	
	5	−0·0048	0·0003		6	0·0024	0·0033	
	7	0·0018	−0·0002		8	−0·0008	−0·0013	
	9	−0·0009	0·0002		10	0·0004	0·0009	
	11	0·0006	−0·0001		12	−0·0003	−0·0005	
2	1	0·2462	0·0016		2	−0·1110	−0·0284	−0·0003
	3	−0·2930	−0·0023		4	−0·0892	0·0226	0·0003
	5	0·0675	0·0005		6	0·0205	−0·0100	−0·0001
	7	−0·0251	0·0004		8	−0·0089	0·0045	
	9	0·0124	−0·0004		10	0·0041	−0·0030	
	11	−0·0080	0·0002		12	−0·0032	0·0017	
3	1	0·0004	0·0005		2	−0·0783	0·0118	
	3	−0·0063	−0·0011		4	−0·0366	−0·0080	
	5	0·0083	0·0007		6	0·0063	0·0022	
	7	−0·0029	−0·0003		8	−0·0026	−0·0008	
	9	0·0014	0·0003		10	0·0011	0·0004	
	11	−0·0009	−0·0001		12	−0·0009	−0·0002	
4	1	−0·0136	0·0009		2	−0·0962	0·0211	0·0003
	3	0·0108	−0·0020		4	−0·0400	−0·0150	−0·0003
	5	0·0037	0·0015		6	0·0058	0·0046	0·0001
	7	−0·0011	−0·0007		8	−0·0024	−0·0017	
	9	0·0006	0·0005		10	0·0011	0·0011	
	11	−0·0004	−0·0002		12	−0·0008	−0·0006	
5	1	−0·0186	0·0030		2	−0·1518	0·0486	0·0008
	3	0·0209	−0·0062		4	−0·0502	−0·0347	−0·0005
	5	−0·0036	0·0044		6	0·0052	0·0113	0·0001
	7	0·0015	−0·0021		8	−0·0022	−0·0045	0·0001
	9	−0·0008	0·0015		10	0·0009	0·0029	−0·0001
	11	0·0006	−0·0006		12	−0·0007	−0·0016	

TABLE XXII B. VALUES OF ψ

		Coefficients of $H/h\,.\cos s\theta\sin n\chi\,.\,e^{i\sigma t}$				Coefficients of $-H/h\,.\,i\sin s\theta\sin n\chi\,.\,e^{i\sigma t}$		
β	s	$n=2$	4	6	s	$n=1$	3	5
6	1	−0·0682	−0·0355	−0·0009	2	0·5329	−0·3529	−0·0059
	3	0·0012	0·0674	0·0017	4	0·0043	0·2484	0·0035
	5	0·0932	−0·0428	−0·0013	6	0·0208	−0·0811	0·0001
	7	−0·0310	0·0189	0·0005	8	−0·0103	0·0341	−0·0013
	9	0·0156	−0·0135		10	0·0049	−0·0218	0·0016
	11	−0·0108	0·0055		12	−0·0043	0·0120	−0·0007
7	1	−0·0451	−0·0082	−0·0002	2	−0·0153	−0·0528	−0·0008
	3	0·0305	0·0149	0·0004	4	−0·0633	0·0353	0·0002
	5	0·0196	−0·0087	−0·0003	6	0·0114	−0·0107	0·0004
	7	−0·0060	0·0035	0·0001	8	−0·0055	0·0050	−0·0004
	9	0·0030	−0·0025		10	0·0025	−0·0033	0·0004
	11	−0·0020	0·0010		12	−0·0021	0·0018	−0·0002
8	1	−0·0593	−0·0102	−0·0003	2	−0·0864	−0·0465	−0·0003
	3	0·0479	0·0182	0·0006	4	−0·1097	0·0288	−0·0004
	5	0·0149	−0·0102	−0·0004	6	0·0149	−0·0075	0·0006
	7	−0·0042	0·0039	0·0001	8	−0·0076	0·0040	−0·0005
	9	0·0022	−0·0028		10	0·0034	−0·0026	0·0005
	11	−0·0015	0·0011		12	−0·0027	0·0014	−0·0002
9	1	−0·1267	−0·0341	−0·0010	2	−0·3063	−0·1163	0·0012
	3	0·1221	0·0610	0·0020	4	−0·3585	0·0679	−0·0035
	5	0·0069	−0·0346	−0·0013	6	0·0380	−0·0153	0·0033
	7	−0·0025	0·0130	0·0002	8	−0·0213	0·0090	−0·0026
	9	0·0018	−0·0088	0·0001	10	0·0095	−0·0060	0·0021
	11	−0·0016	0·0035		12	−0·0078	0·0034	−0·0007
10	1	0·0034	0·0273	0·0009	2	0·1627	0·0686	−0·0031
	3	−0·0283	−0·0503	−0·0017	4	0·2175	−0·0417	0·0050
	5	0·0290	0·0303	0·0010	6	−0·0157	0·0111	−0·0038
	7	−0·0052	−0·0120	−0·0001	8	0·0110	−0·0063	0·0027
	9	0·0021	0·0077	−0·0001	10	−0·0048	0·0041	−0·0021
	11	−0·0010	−0·0030		12	0·0039	−0·0023	0·0007

Table XXII c. Values of ψ

	Coefficients of $H/h\,.\,\cos s\theta \sin n\chi\,.\,e^{i\sigma t}$				Coefficients of $-H/h\,.\,i \sin s\theta \sin n\chi\,.\,e^{i\sigma t}$			
β	s	$n=2$	4	6	s	$n=1$	3	5
11	1	−0·0597	0·0155	0·0005	2	0·0224	0·0205	−0·0037
	3	0·0342	−0·0298	−0·0010	4	0·0553	−0·0175	0·0049
	5	0·0286	0·0193	0·0006	6	−0·0005	0·0085	−0·0032
	7	−0·0040	−0·0081		8	0·0019	−0·0038	0·0020
	9	0·0017	0·0051	−0·0001	10	−0·0009	0·0024	−0·0015
	11	−0·0008	−0·0020		12	0·0008	−0·0013	0·0005
12	1	−0·1661	0·0275	0·0008	2	−0·0441	−0·0077	−0·0116
	3	0·1258	−0·0541	−0·0015	4	−0·0304	−0·0154	0·0135
	5	0·0413	0·0363	0·0008	6	0·0071	0·0201	−0·0077
	7	−0·0016	−0·0157	0·0003	8	−0·0025	−0·0074	0·0045
	9	0·0005	0·0100	−0·0005	10	0·0010	0·0048	−0·0035
	11	0·0001	−0·0040	0·0001	12	−0·0006	−0·0027	0·0012
13	1	0·3284	−0·0667	−0·0012	2	0·0659	0·1122	0·0404
	3	−0·3086	0·1280	0·0025	4	0·2365	−0·0057	−0·0438
	5	−0·0038	−0·0824	−0·0009	6	−0·0094	−0·0493	0·0230
	7	−0·0186	0·0350	−0·0018	8	0·0092	0·0157	−0·0126
	9	0·0096	−0·0232	0·0018	10	−0·0038	−0·0107	0·0098
	11	−0·0070	0·0093	−0·0004	12	0·0024	0·0063	−0·0034
14	1	0·0509	−0·0265	−0·0002	2	−0·0507	0·0593	0·0192
	3	−0·0802	0·0480	0·0005	4	0·0872	−0·0145	−0·0203
	5	0·0405	−0·0279	0·0002	6	0·0046	−0·0143	0·0103
	7	−0·0134	0·0111	−0·0011	8	0·0013	0·0043	−0·0054
	9	0·0068	−0·0079	0·0009	10	−0·0004	−0·0032	0·0041
	11	−0·0046	0·0032	−0·0003	12	0·0002	0·0019	−0·0014
15	1	−0·0400	−0·0446		2	−0·2115	0·0927	0·0354
	3	−0·0568	0·0765	−0·0002	4	0·0846	−0·0340	−0·0375
	5	0·1173	−0·0393	0·0014	6	0·0199	−0·0118	0·0192
	7	−0·0249	0·0141	−0·0028	8	−0·0026	0·0035	−0·0099
	9	0·0132	−0·0113	0·0021	10	0·0014	−0·0033	0·0072
	11	−0·0088	0·0046	−0·0005	12	−0·0012	0·0022	−0·0024

Table XXII d. Values of ψ

		Coefficients of $H/h\,.\cos s\theta\sin n\chi\,.\,e^{i\sigma t}$				Coefficients of $-H/h\,.\,i\sin s\theta\sin n\chi\,.\,e^{i\sigma t}$		
β	s	$n=2$	4	6	s	$n=1$	3	5
16	1	0·1686	0·0597	−0·0008	2	0·3880	−0·0855	−0·0507
	3	0·0004	−0·0982	0·0019	4	−0·0272	0·0457	0·0549
	5	−0·1919	0·0455	−0·0035	6	−0·0371	−0·0011	−0·0288
	7	0·0288	−0·0149	0·0051	8	0·0084	−0·0006	0·0148
	9	−0·0164	0·0135	−0·0035	10	−0·0041	0·0020	−0·0107
	11	0·0105	−0·0056	0·0008	12	0·0034	−0·0017	0·0036
17	1	0·0594	0·0148	−0·0005	2	0·1085	−0·0066	−0·0139
	3	−0·0111	−0·0240	0·0011	4	0·0091	0·0092	0·0155
	5	−0·0521	0·0106	−0·0015	6	−0·0109	−0·0044	−0·0086
	7	0·0050	−0·0034	0·0018	8	0·0030	0·0005	0·0044
	9	−0·0034	0·0034	−0·0012	10	−0·0014		−0·0031
	11	0·0022	−0·0014	0·0003	12	0·0011	−0·0001	0·0011
18	1	0·0370	0·0077	−0·0006	2	0·0575	0·0084	−0·0088
	3	−0·0090	−0·0127	0·0013	4	0·0099	0·0022	0·0103
	5	−0·0286	0·0059	−0·0014	6	−0·0067	−0·0049	−0·0058
	7	0·0010	−0·0022	0·0014	8	0·0019	0·0006	0·0031
	9	−0·0013	0·0021	−0·0009	10	−0·0008	−0·0001	−0·0022
	11	0·0009	−0·0008	0·0002	12	0·0006		0·0007
19	1	0·0253	0·0043	−0·0010	2	0·0309	0·0196	−0·0075
	3	−0·0065	−0·0079	0·0018	4	0·0066	−0·0022	0·0093
	5	−0·0178	0·0047	−0·0017	6	−0·0055	−0·0054	−0·0056
	7	−0·0009	−0·0023	0·0016	8	0·0013	0·0005	0·0030
	9	−0·0003	0·0020	−0·0010	10	−0·0006	−0·0002	−0·0021
	11	0·0002	−0·0008	0·0003	12	0·0005		0·0007
20	1	0·0165	0·0010	−0·0018	2	0·0053	0·0377	−0·0087
	3	−0·0046	−0·0042	0·0035	4	−0·0006	−0·0089	0·0113
	5	−0·0093	0·0052	−0·0029	6	−0·0059	−0·0068	−0·0073
	7	−0·0028	−0·0036	0·0023	8	0·0010	0·0003	0·0040
	9	0·0005	0·0027	−0·0014	10	−0·0005	0·0001	−0·0028
	11	−0·0003	−0·0011	0·0003	12	0·0002	−0·0002	0·0010

TABLE XXIII A. VALUES OF $\zeta/He^{i\sigma t}$ FOR THE SEMIDIURNAL TIDE (K_2)

		Coefficients of $\cos s\theta \cos n\chi$				Coefficients of $-i \sin s\theta \cos n\chi$ ($s > 0$) Coefficients of $-i\theta \cos n\chi$ ($s = 0$)			
β	s	$n = 0$	2	4	6	$n = 1$	3	5	7
1	0	0·4666	−0·5694	0·0036		−0·0702	0·0698	0·0004	
	2	0·8659	0·2020	0·0002		−0·9431	−0·0567	−0·0002	
	4	0·0018	0·0017	−0·0004		−0·0125	0·0126	−0·0001	
	6	−0·0003	−0·0007	0·0001		0·0016	−0·0017	0·0001	
	8	0·0001	0·0002	−0·0001		−0·0007	0·0008	−0·0001	
	10		−0·0002	0·0001		0·0004	−0·0004	0·0001	
	12					−0·0001	0·0002		
	1	−0·3621	0·3696	−0·0075		−0·5469	−0·0408		
	3	−0·0045	−0·0018	0·0063		−0·0348	0·0159		
	5	0·0043	−0·0011	−0·0032		0·0032	−0·0025		
	7	−0·0008	−0·0003	0·0011		−0·0006	0·0006		
	9	0·0003	0·0002	−0·0005		0·0003	−0·0002		
	11	−0·0002	−0·0002	0·0004		−0·0001	0·0003		
	13	0·0001		−0·0001					
2	0	−1·470	3·259	0·007		0·323	−0·320	−0·003	
	2	2·066	1·014	−0·011		−1·392	0·388	0·004	
	4	−0·043	−0·067	−0·002		0·159	−0·157	−0·001	
	6	0·009	0·019			−0·044	0·044		
	8	−0·003	−0·001			0·018	−0·019		
	10	0·001	0·005			−0·010	0·010		
	12	−0·001	0·001			0·004	−0·004		
	1	4·248	−4·244	−0·005		9·470	0·173	0·002	
	3	−0·041	0·066	−0·024		−0·011	−0·063	−0·001	
	5	0·049	−0·067	0·017		0·007	0·010		
	7	−0·014	0·021	−0·007			−0·003		
	9	0·006	−0·010	0·004		0·001	0·001		
	11	−0·004	0·006	−0·002			0·001		
	13	0·001	−0·002	0·001					

Table XXIII b. Values of $\zeta/He^{i\sigma t}$ for the semidiurnal tide (K_2)

		Coefficients of $\cos s\theta \cos n\chi$				Coefficients of $-i \sin s\theta \cos n\chi$ ($s>0$) Coefficients of $-i\theta \cos n\chi$ ($s=0$)			
β	s	$n=0$	2	4	6	$n=1$	3	5	7
3	0	0·385	−0·166	0·028		−0·155	0·153	0·001	
	2	1·584	0·412	0·007		−0·928	−0·073		
	4	−0·002	−0·021	−0·003		0·010	−0·009	−0·001	
	6	0·002	0·004			−0·008	0·007	0·001	
	8	−0·001	−0·001	−0·001		0·003	−0·003		
	10		0·001			−0·002	0·001		
	12					0·001	−0·001		
	1	0·254	−0·212	−0·042		1·409	−0·081		
	3	−0·019	0·002	0·017		−0·066	0·031		
	5	0·029	−0·022	−0·007		0·007	−0·004		
	7	−0·006	0·004	0·002		−0·001	0·001		
	9	0·003	−0·002	−0·001		0·001			
	11	−0·002	0·001						
	13	0·001							
4	0	0·739	−0·717	0·070		−0·394	0·390	0·005	
	2	2·038	0·538	0·017		−0·778	−0·222	0·001	
	4	0·009	−0·028	−0·008		−0·008	0·012	−0·004	
	6	0·001	0·003	0·001		−0·004	0·002	0·003	
	8		−0·001	−0·001		0·002		−0·001	
	10		0·001	0·001		−0·001		0·001	
	12								
	1	−0·147	0·257	−0·110	−0·001	1·152	−0·182	−0·004	
	3	−0·011	−0·034	0·045	0·001	−0·034	0·071	0·003	
	5	0·040	−0·021	−0·019		0·005	−0·011	−0·001	
	7	−0·008	0·002	0·006			0·003		
	9	0·003	−0·001	−0·003		0·001	−0·001		
	11	−0·002		0·002			0·001		
	13	0·001		−0·001					

TABLE XXIII c. VALUES OF $\zeta/He^{i\sigma t}$ FOR THE SEMIDIURNAL TIDE (K_2)

		Coefficients of $\cos s\theta \cos n\chi$				Coefficients of $-i \sin s\theta \cos n\chi$ $(s>0)$ Coefficients of $-i\theta \cos n\chi$ $(s=0)$			
β	s	$n=0$	2	4	6	$n=1$	3	5	7
5	0	1·341	−1·953	0·268	0·006	−1·164	1·150	0·013	
	2	3·159	1·256	0·068		−0·353	−0·652	0·006	
	4	0·040	−0·046	−0·029		−0·039	0·054	−0·014	
	6	−0·001	0·002	0·004		0·007	−0·016	0·009	
	8	0·001	−0·002	−0·003		−0·003	0·008	−0·005	
	10			0·001		0·002	−0·005	0·003	
	12					−0·001	0·002	−0·001	
	1	−0·536	0·950	−0·405	−0·008	1·675	−0·466	−0·018	
	3	0·057	−0·199	0·139	0·003	0·240	0·178	0·005	
	5	0·057		−0·057	−0·001	−0·015	−0·029	−0·001	
	7	−0·008	−0·011	0·020		0·005	0·008	0·001	
	9	0·004	0·006	−0·010		−0·001	−0·002	0·001	
	11	−0·002	−0·004	0·006		0·001	0·003		
	13	0·001	0·001	−0·002					
6	0	−2·95	13·59	−3·21	−0·06	10·15	−10·07	−0·07	
	2	−7·00	−11·15	−0·89	−0·03	− 6·23	5·37	−0·13	
	4	−0·29		0·32	0·01	0·27	− 0·46	0·18	
	6	0·03		−0·03	−0·01	− 0·19	0·29	−0·10	
	8	−0·01	0·02	0·03		0·07	− 0·12	0·05	
	10		0·01	−0·02		− 0·04	0·07	−0·03	
	12		0·01			0·02	− 0·02	0·01	
	1	−0·14	−4·74	4·77	0·11	−10·06	3·26	0·12	
	3	−1·30	2·66	−1·34	−0·02	− 4·74	− 1·27	−0·05	
	5	0·06	−0·56	0·49		0·31	0·26		
	7	−0·05	0·23	−0·18		− 0·07	− 0·06		
	9	0·02	−0·11	0·09		0·03	0·01		
	11	−0·01	0·06	−0·05		− 0·01	− 0·03		
	13		−0·02	0·02					

TABLE XXIII D. VALUES OF $\zeta/He^{i\sigma t}$ FOR THE SEMIDIURNAL TIDE (K_2)

		Coefficients of $\cos s\theta \cos n\chi$				Coefficients of $-i \sin s\theta \cos n\chi$ $(s>0)$ Coefficients of $-i\theta \cos n\chi$ $(s=0)$			
β	s	$n=0$	2	4	6	$n=1$	3	5	7
7	0	1·250	1·513	−0·775	−0·020	1·721	−1·727	0·006	
	2	2·165	−2·180	−0·225	−0·008	−1·779	0·830	−0·049	−0·001
	4	0·011	−0·084	0·076	0·004	0·007	−0·054	0·046	0·001
	6	0·009	−0·002	−0·006	−0·001	−0·045	0·069	−0·023	−0·001
	8	−0·002	0·005	0·006		0·018	−0·026	0·015	
	10	0·001	0·002	−0·003		−0·009	0·015	−0·006	
	12	−0·001	0·001	−0·002		0·003	−0·005	0·002	
	1	−1·455	0·295	1·130	0·029	−1·790	0·388	0·018	
	3	−0·346	0·609	−0·259	−0·004	−1·030	−0·154	−0·006	
	5	0·121	−0·200	0·080	−0·001	0·066	0·034	−0·002	
	7	−0·029	0·059	−0·031	0·002	−0·013	−0·008	0·001	
	9	0·014	−0·029	0·017	−0·002	0·007		−0·001	
	11	−0·008	0·017	−0·011	0·001	−0·002	−0·004	0·001	
	13	0·003	−0·006	0·003					
8	0	2·901	0·658	−1·065	−0·026	1·641	−1·680	0·039	0·001
	2	4·902	−2·374	−0·300	−0·014	−1·635	0·726	−0·089	−0·002
	4	0·140	−0·150	0·098	0·007	−0·049	−0·018	0·066	0·002
	6	0·007	−0·014	−0·007	−0·002	−0·040	0·072	−0·031	−0·001
	8	−0·002	0·006	0·008		0·013	−0·028	0·014	
	10	0·001	0·001	−0·004		−0·008	0·016	−0·008	
	12	−0·001	0·002	−0·002		0·003	−0·005	0·002	
	1	−2·988	1·438	1·511	0·039	−2·276	0·166	0·009	
	3	−0·377	0·676	−0·295	−0·003	−0·657	−0·066	0·005	
	5	0·219	−0·290	0·075	−0·003	0·051	0·021	−0·004	
	7	−0·045	0·070	−0·028	0·003	−0·007	−0·005		
	9	0·022	−0·035	0·016	−0·002	0·007		−0·001	
	11	−0·012	0·020	−0·010	0·001		−0·004	0·001	
	13	0·005	−0·008	0·003					

TABLE XXIII E. VALUES OF $\zeta/He^{i\sigma t}$ FOR THE SEMIDIURNAL TIDE (K_2)

		Coefficients of $\cos s\theta \cos n\chi$				Coefficients of $-i \sin s\theta \cos n\chi$ $(s>0)$ Coefficients of $-i\theta \cos n\chi$ $(s=0)$			
β	s	$n=0$	2	4	6	$n=1$	3	5	7
9	0	10·89	0·48	−4·03	−0·11	4·33	−4·61	0·27	0·01
	2	17·47	−6·26	−1·04	−0·05	−2·56	1·99	−0·42	−0·02
	4	0·93	−0·33	0·35	0·02	−0·26	−0·01	0·26	0·01
	6		−0·10	−0·03	−0·01	−0·04	0·16	−0·12	
	8	0·01	0·03	0·03		0·01	−0·07	0·05	
	10		−0·01	−0·01		−0·01	0·04	−0·03	
	12		0·01	−0·01			−0·01	0·01	
	1	−11·03	5·31	5·57	0·15	−7·66	−0·37	−0·04	
	3	− 0·69	1·69	−0·99		1·58	0·18	0·06	
	5	0·76	−0·95	0·21	−0·02	0·06		−0·03	
	7	− 0·13	0·19	−0·08	0·02	0·01	−0·01		
	9	0·07	−0·10	0·05	−0·01	0·01		−0·01	
	11	− 0·04	0·06	−0·03	0·01		−0·01		
	13	0·02	−0·02	0·01					
10	0	− 7·23	−0·85	3·85	0·09	−2·73	3·14	−0·40	−0·02
	2	−10·88	2·93	0·81	0·05	0·20	−1·70	0·49	0·02
	4	− 1·08	−0·21	−0·30	−0·02	0·15	0·11	−0·26	−0·01
	6	0·02	0·13	0·04		−0·06	−0·06	0·12	
	8	− 0·01	−0·03	−0·03		0·01	0·04	−0·05	
	10		0·01	0·01		−0·01	−0·02	0·03	
	12		−0·01	0·01			0·01	−0·01	
	1	7·09	−1·82	−5·15	−0·12	5·06	1·28	0·12	
	3	− 0·17	−0·72	0·90	−0·01	−5·15	−0·56	−0·09	
	5	− 0·49	0·64	−0·18	0·03	−0·02	0·06	0·04	
	7	0·07	−0·11	0·07	−0·02	−0·02	0·01	−0·01	
	9	− 0·04	0·07	−0·04	0·01			0·01	
	11	0·02	−0·04	0·02	−0·01	−0·01	0·01		
	13	− 0·01	0·01	−0·01					

TABLE XXIII F. VALUES OF $\zeta/He^{i\sigma t}$ FOR THE SEMIDIURNAL TIDE (K_2)

β	s	Coefficients of $\cos s\theta \cos n\chi$				Coefficients of $-i\sin s\theta\cos n\chi$ $(s>0)$ Coefficients of $-i\theta\cos n\chi$ $(s=0)$			
		$n=0$	2	4	6	$n=1$	3	5	7
11	0	−1·916	−1·582	−2·694	−0·060	−0·923	1·344	−0·406	0·015
	2	−2·519	−0·051	0·398	0·264	−0·271	−1·157	0·413	0·015
	4	−0·682	−0·587	−0·189	−0·143	−0·049	0·239	−0·185	−0·006
	6	0·028	0·091	0·030	0·002	−0·067	−0·018	0·083	0·001
	8	−0·011	−0·020	−0·018	0·001	0·013	0·026	−0·040	0·001
	10	0·002	0·010	0·008	0·001	−0·006	−0·015	0·021	−0·001
	12	−0·002	−0·006	0·006		0·002	0·004	−0·006	
	1	1·542	1·937	−3·419	−0·059	1·099	1·663	0·142	
	3	−0·570		0·590	−0·021	−4·804	−0·677	−0·103	
	5	−0·129	0·236	−0·135	0·027	−0·060	0·059	0·036	
	7	0·009	−0·044	0·054	−0·018	−0·009	0·003	−0·005	
	9	−0·007	0·026	−0·030	0·011	0·001	0·003	0·006	
	11	0·005	−0·015	0·016	−0·006	−0·003	0·002	−0·003	
	13	−0·002	0·005	−0·005	0·001				
12	0	1·28	−4·92	5·67	0·05	0·25	0·99	−1·19	−0·05
	2	2·59	−3·22	0·50	0·04	0·07	−2·18	1·06	0·04
	4	−0·88	−1·84	−0·34	−0·02	−0·47	0·87	−0·40	−0·01
	6	0·06	0·13	0·07		−0·09	−0·09	0·17	
	8	−0·01	−0·04	−0·03			0·08	−0·09	
	10		0·02	0·02			−0·05	0·05	
	12		−0·01	0·01			0·01	−0·01	
	1	−2·47	9·34	−6·83	−0·04	−2·73	5·21	0·48	
	3	−1·48	0·45	1·12	−0·08	−8·50	−1·96	−0·31	
	5	0·07	0·13	−0·28	0·08	−0·25	0·10	0·10	
	7	−0·03	−0·06	0·13	−0·05	0·01		−0·01	
	9	0·01	0·03	−0·07	0·03		0·01	0·01	
	11		−0·02	0·04	−0·01		0·01	−0·01	
	13		0·01	−0·01					

TABLE XXIII G. VALUES OF $\zeta/He^{i\sigma t}$ FOR THE SEMIDIURNAL TIDE (K_2)

		Coefficients of $\cos s\theta \cos n\chi$				Coefficients of $-i \sin s\theta \cos n\chi$ ($s>0$) Coefficients of $-i\theta \cos n\chi$ ($s=0$)			
β	s	$n=0$	2	4	6	$n=1$	3	5	7
13	0	−9·10	12·94	−14·99		−5·99	1·73	4·08	0·18
	2	−14·15	11·57	−0·69	−0·01	−1·65	4·16	−3·38	−0·13
	4	0·37	4·70	0·71	0·03	1·85	−2·94	1·07	0·02
	6	−0·12	−0·13	−0·17		−0·11	0·53	−0·43	0·01
	8	0·01	0·10	0·08	−0·01	0·10	−0·31	0·23	−0·02
	10		−0·01	−0·04		−0·06	0·17	−0·12	0·01
	12		0·03	0·02		0·02	−0·05	0·03	
	1	11·87	−28·87	17·19	−0·19	15·07	−17·16	−1·77	
	3	2·83	−0·67	−2·50	0·35	9·62	5·97	1·08	
	5	−0·58	0·20	0·65	−0·26	0·68	−0·10	−0·31	
	7	0·05	0·17	−0·37	0·14	−0·10	0·03	0·03	
	9	−0·04	−0·06	0·18	−0·08	0·03	−0·04	−0·04	
	11	0·02	0·04	−0·10	0·04	−0·02	−0·01	0·03	
	13	−0·01	−0·01	0·03	−0·01				
14	0	−3·00	3·86	−5·97	0·07	−3·92	1·82	2·00	0·09
	2	−4·28	4·25	−0·13	0·02	−0·22	0·88	−1·60	−0·06
	4	−0·59	1·16	0·19	0·01	0·78	−1·23	0·44	0·01
	6	−0·02	0·02	−0·05	0·01	−0·19	0·34	−0·16	0·01
	8	−0·01	0·03	0·03	−0·01	0·07	−0·15	0·08	−0·01
	10		0·01	−0·01		−0·04	0·08	−0·05	0·01
	12		0·01	−0·01		0·02	−0·03	0·01	
	1	3·44	−9·89	6·68	−0·22	6·41	−7·39	−0·88	
	3	0·31	0·32	−0·83	0·20	−2·17	2·40	0·52	
	5	−0·22	0·15	0·20	−0·13	0·05	0·03	−0·14	
	7	−0·01	0·08	−0·14	0·07	−0·03	0·03	0·01	
	9	−0·01	−0·02	0·07	−0·04	0·01	−0·02	−0·02	
	11		0·02	−0·04	0·02			0·01	
	13			0·01					

TABLE XXIII H. VALUES OF $\zeta/He^{i\sigma t}$ FOR THE SEMIDIURNAL TIDE (K_2)

		Coefficients of $\cos s\theta \cos n\chi$				Coefficients of $-i \sin s\theta \cos n\chi$ $(s>0)$ Coefficients of $-i\theta \cos n\chi$ $(s=0)$			
β	s	$n=0$	2	4	6	$n=1$	3	5	7
15	0	−1·56	3·61	−10·12	0·35	−7·85	3·73	3·94	0·18
	2	−1·81	5·31	0·03	0·08	1·89	0·33	−3·10	−0·12
	4	−1·99	0·05	0·14	−0·02	1·08	−1·89	0·81	
	6	−0·01	0·09	−0·01	0·02	−0·48	0·73	−0·25	0·03
	8	−0·02	0·05	0·03	−0·02	0·15	−0·26	0·14	−0·02
	10	0·01	0·02	−0·02		−0·08	0·15	−0·08	0·01
	12		0·02	−0·02		0·03	−0·05	0·02	
	1	0·79	−11·19	11·06	−0·66	8·04	−11·83	−1·64	
	3	−0·90	1·65	−1·19	0·44	−12·12	3·62	0·98	
	5	−0·22	0·25	0·24	−0·28	−0·63	0·11	−0·27	
	7	−0·08	0·16	−0·21	0·13	0·01	0·10	0·02	
	9	0·01	−0·05	0·11	−0·07		−0·04	−0·04	
	11	−0·01	0·04	−0·06	0·04		0·01	0·02	
	13		−0·01	0·02	−0·01				
16	0	−2·94	−1·67	13·81	−0·79	10·67	−4·31	−6·08	−0·29
	2	−4·72	−4·64	−0·42	−0·20	−6·48	0·53	4·78	0·17
	4	3·83	2·92	0·11	0·06	−0·91	2·10	−1·21	0·02
	6		−0·09	−0·07	−0·04	0·80	−1·10	0·35	−0·05
	8	0·03	−0·05	−0·04	0·03	−0·19	0·32	−0·18	0·05
	10	−0·01	−0·03	0·02		0·11	−0·19	0·11	−0·02
	12	0·01	−0·04	0·02		−0·04	0·07	−0·03	
	1	5·98	7·55	−14·91	1·39	−4·86	13·43	2·31	
	3	2·69	−3·50	1·56	−0·76	23·84	−3·97	−1·41	
	5	0·10	−0·29	−0·26	0·45	2·12	−0·10	0·41	
	7	0·16	−0·20	0·26	−0·21	−0·13	−0·20	−0·04	
	9	−0·05	0·07	−0·14	0·12	0·04	0·07	0·06	
	11	0·03	−0·05	0·08	−0·06	−0·02	−0·02	−0·03	
	13	−0·01	0·02	−0·02	0·01				

TABLE XXIII I. VALUES OF $\zeta/He^{i\sigma t}$ FOR THE SEMIDIURNAL TIDE (K_2)

		Coefficients of $\cos s\theta \cos n\chi$				Coefficients of $-i \sin s\theta \cos n\chi$ $(s>0)$ Coefficients of $-i\theta \cos n\chi$ $(s=0)$			
β	s	$n=0$	2	4	6	$n=1$	3	5	7
17	0	−1·807	0·071	3·713	−0·331	2·446	−0·540	−1·821	−0·085
	2	−2·574	−0·729	−0·294	−0·087	−2·696	0·212	1·440	0·045
	4	1·307	1·601	0·128	0·031	−0·046	0·399	−0·364	0·011
	6	0·009	0·003	−0·048	−0·017	0·217	−0·298	0·102	−0·021
	8	0·007	−0·014	−0·009	0·011	−0·038	0·073	−0·052	0·017
	10	−0·003	−0·006	0·004	−0·001	0·024	−0·047	0·030	−0·008
	12	0·003	−0·005	0·009		−0·009	0·016	−0·008	0·001
	1	3·041	0·341	−3·933	0·552	0·517	2·434	0·609	
	3	0·947	−1·158	0·466	−0·255	7·205	−0·741	−0·374	
	5	0·002	−0·069	−0·075	0·142	0·945	0·037	0·119	
	7	0·050	−0·046	0·063	−0·068	−0·071	−0·070	−0·016	
	9	−0·017	0·018	−0·039	0·037	0·021	0·020	0·016	
	11	0·010	−0·012	0·020	−0·018	−0·014	−0·008	−0·010	
	13	−0·004	0·005	−0·005	0·004				
18	0	−1·425	0·136	2·287	−0·324	1·020	0·297	−1·260	−0·057
	2	−1·887	−0·288	−0·371	−0·086	−2·079	0·051	1·013	0·016
	4	1·017	1·492	0·160	0·036	0·117	0·130	−0·263	0·017
	6	0·022	0·031	−0·045	−0·014	0·119	−0·175	0·074	−0·018
	8	0·002	−0·007	−0·005	0·010	−0·012	0·036	−0·037	0·013
	10		−0·003	0·001	0·001	0·010	−0·025	0·021	−0·006
	12		−0·002	0·004	−0·004	−0·004	0·008	−0·006	0·002
	1	2·354	−0·516	−2·345	0·507	1·427	0·506	0·315	
	3	0·613	−0·780	0·364	−0·198	4·297	−0·230	−0·234	
	5	0·006	−0·039	−0·071	0·104	0·817	0·100	0·081	
	7	0·032	−0·020	0·037	−0·049	−0·064	−0·054	−0·011	
	9	−0·011	0·010	−0·027	0·028	0·021	0·014	0·015	
	11	0·006	−0·006	0·014	−0·014	−0·011	−0·005	−0·006	
	13	−0·002	0·002	−0·003	0·003				

TABLE XXIII J. VALUES OF $\zeta/He^{i\sigma t}$ FOR THE SEMIDIURNAL TIDE (K_2)

		Coefficients of $\cos s\theta \cos n\chi$				Coefficients of $-i \sin s\theta \cos n\chi\ (s>0)$ Coefficients of $-i\theta \cos n\chi\ \ (s=0)$			
β	s	$n=0$	2	4	6	$n=1$	3	5	7
19	0	−1·130	−0·036	1·906	−0·408	0·227	1·017	−1·203	−0·041
	2	−1·336	−0·227	−0·579	−0·131	−1·826	−0·147	0·962	0·010
	4	1·074	1·653	0·224	0·042	0·241	−0·005	−0·256	0·019
	6	0·038	0·056	−0·050	−0·023	0·073	−0·130	0·079	−0·021
	8	−0·002	−0·007	−0·004	0·010	0·001	0·025	−0·041	0·016
	10		0·002	0·001	−0·001	0·003	−0·018	0·023	−0·008
	12		−0·004	0·004	−0·002	−0·001	0·005	−0·006	0·002
	1	1·948	−0·767	−1·836	0·656	2·046	−0·739	0·184	
	3	0·431	−0·638	0·421	−0·213	2·893	0·015	−0·178	
	5	0·026	−0·025	−0·105	0·103	0·861	0·196	0·080	
	7	0·025	−0·011	0·035	−0·050	−0·065	−0·052	−0·013	
	9	−0·007	0·007	−0·028	0·028	0·022	0·016	0·012	
	11	0·005	−0·005	0·014	−0·014	−0·013	−0·006	−0·006	
	13	−0·002	0·002	−0·003	0·003				
20	0	−0·767	−0·387	2·039	−0·771	−0·724	2·317	−1·566	−0·027
	2	−0·598	−0·340	−1·122	−0·226	−1·701	−0·535	1·252	−0·016
	4	1·522	2·232	0·388	0·090	0·458	−0·151	−0·347	0·040
	6	0·080	0·103	−0·071	−0·030	0·034	−0·118	0·119	−0·035
	8	−0·006	−0·010	−0·004	0·019	0·014	0·024	−0·061	0·023
	10	0·002	0·006		0·001	−0·004	−0·018	0·032	−0·010
	12	−0·004	−0·002	0·006	−0·002	0·001	0·005	−0·008	0·002
	1	1·579	−0·992	−1·729	1·142	3·072	−2·524	0·078	
	3	0·250	−0·606	0·675	−0·319	1·705	0·276	−0·155	
	5	0·071	−0·001	−0·209	0·139	1·078	0·408	0·104	
	7	0·026	−0·008	0·052	−0·069	−0·078	−0·065	−0·021	
	9	−0·006	0·009	−0·043	0·041	0·023	0·021	0·014	
	11	0·003	−0·006	0·023	−0·020	−0·013	−0·008	−0·010	
	13	−0·001	0·002	−0·005	0·005				

Table XXIV a. Values of ζ_1/H and ζ_2/H for the semidiurnal tide (K_2)

		ζ_1/H					ζ_2/H				
β	χ	$\theta=20°$	40°	60°	80°	90°	20°	40°	60°	80°	90°
0	0°	0·766	0·174	−0·500	−0·940	−1·000	−0·643	−0·985	−0·866	−0·342	0·000
	20	0·780	0·222	−0·412	−0·826	−0·883	−0·604	−0·925	−0·814	−0·321	0·000
	40	0·814	0·344	−0·190	−0·539	−0·587	−0·492	−0·754	−0·663	−0·262	0·000
	60	0·854	0·484	0·063	−0·212	−0·250	−0·321	−0·492	−0·443	−0·171	0·000
	80	0·880	0·574	0·227	0·001	−0·030	−0·112	−0·171	−0·150	−0·059	0·000
	90	0·883	0·587	0·250	0·030	0·000	0·000	0·000	0·000	0·000	0·000
1	0	0·720	0·084	−0·636	−1·101	−1·163	−0·859	−1·379	−1·376	−0·904	−0·569
	20	0·736	0·144	−0·516	−0·934	−0·986	−0·806	−1·290	−1·285	−0·855	−0·556
	40	0·775	0·294	−0·221	−0·517	−0·533	−0·654	−1·039	−1·030	−0·708	−0·498
	60	0·821	0·462	0·104	−0·052	−0·014	−0·425	−0·669	−0·660	−0·470	−0·358
	80	0·850	0·569	0·308	0·245	0·328	−0·147	−0·230	−0·226	−0·165	−0·132
	90	0·854	0·584	0·336	0·285	0·375	0·000	0·000	0·000	0·000	0·000
2	0	4·128	2·425	0·333	−1·209	−1·445	2·606	5·149	7·466	9·235	9·740
	20	4·116	2·359	0·171	−1·561	−1·944	2·437	4·775	6·903	8·637	9·267
	40	4·085	2·197	−0·215	−2·419	−3·213	1·962	3·765	5·399	6·958	7·786
	60	4·046	2·019	−0·612	−3·339	−4·667	1·263	2·366	3·360	4·483	5·251
	80	4·018	1·906	−0·846	−3·907	−5·622	0·434	0·802	1·131	1·544	1·862
	90	4·014	1·891	−0·877	−3·983	−5·753	0·000	0·000	0·000	0·000	0·000
3	0	1·776	0·614	−0·734	−1·657	−1·792	−0·217	−0·165	0·281	0·996	1·366
	20	1·786	0·668	−0·613	−1·505	−1·664	−0·203	−0·155	0·246	0·890	1·227
	40	1·813	0·802	−0·319	−1·124	−1·325	−0·165	−0·128	0·166	0·636	0·887
	60	1·850	0·950	−0·006	−0·700	−0·910	−0·107	−0·087	0·086	0·354	0·499
	80	1·876	1·043	0·185	−0·428	−0·622	−0·037	−0·031	0·031	0·110	0·156
	90	1·880	1·056	0·210	−0·391	−0·582	0·000	0·000	0·000	0·000	0·000
4	0	2·074	0·566	−1·185	−2·369	−2·537	−0·285	−0·329	−0·023	0·569	0·915
	20	2·093	0·660	−0·963	−2·070	−2·271	−0·266	−0·312	−0·052	0·433	0·709
	40	2·141	0·887	−0·440	−1·337	−1·556	−0·212	−0·258	−0·098	0·153	0·265
	60	2·197	1·129	0·092	−0·536	−0·669	−0·135	−0·167	−0·098	−0·036	−0·058
	80	2·235	1·276	0·406	−0·030	−0·049	−0·046	−0·057	−0·041	−0·041	−0·073
	90	2·240	1·296	0·447	0·039	0·039	0·000	0·000	0·000	0·000	0·000
5	0	3·091	0·471	−2·554	−4·582	−4·866	0·094	0·153	0·212	0·431	0·701
	20	3·134	0·696	−2·032	−3·878	−4·247	0·093	0·125	0·087	0·078	0·171
	40	3·248	1·237	−0·805	−2·124	−2·498	0·083	0·061	−0·146	−0·578	−0·874
	60	3·384	1·806	0·428	−0·198	−0·222	0·056	0·008	−0·239	−0·813	−1·329
	80	3·476	2·154	1·143	1·018	1·408	0·019	−0·005	−0·112	−0·374	−0·635
	90	3·489	2·201	1·236	1·182	1·640	0·000	0·000	0·000	0·000	0·000

TABLE XXIV B. VALUES OF ζ_1/H AND ζ_2/H FOR THE SEMIDIURNAL TIDE (K_2)

		ζ_1/H					ζ_2/H				
β	χ	$\theta=20°$	40°	60°	80°	90°	20°	40°	60°	80°	90°
6	0°	−7·25	4·05	16·88	25·32	26·55	−7·68	−10·61	−7·19	−1·26	0·17
	20	−7·70	2·13	12·87	20·30	22·36	−7·19	−9·44	−5·21	2·19	4·90
	40	−8·80	−2·41	3·61	7·72	9·83	−5·81	−6·65	−1·31	8·34	13·87
	60	−10·04	−7·12	−5·50	−6·43	−7·71	−3·75	−3·64	1·04	9·86	16·48
	80	−10·92	−10·00	−10·69	−15·62	−21·01	−1·29	−1·12	0·73	4·33	7·43
	90	−11·05	−10·39	−11·36	−16·88	−22·95	0·00	0·00	0·00	0·00	0·00
7	0	1·771	1·926	2·083	2·209	2·230	−2·057	−2·927	−2·155	−0·614	−0·066
	20	1·675	1·579	1·480	1·566	1·776	−1·913	−2·588	−1·636	0·130	0·841
	40	1·431	0·745	0·079	−0·142	0·148	−1·522	−1·795	−0·600	1·467	2·592
	60	1·155	−0·123	−1·308	−2·189	−2·501	−0·965	−0·959	0·085	1·929	3·155
	80	0·974	−0·640	−2·106	−3·562	−4·664	−0·328	−0·290	0·126	0·837	1·437
	90	0·949	−0·708	−2·209	−3·751	−4·986	0·000	0·000	0·000	0·000	0·000
8	0	4·180	2·778	1·291	0·473	0·378	−1·921	−2·970	−2·752	−1·740	−1·294
	20	4·062	2·415	0·784	0·083	0·240	−1·779	−2·577	−2·059	−0·794	−0·236
	40	3·760	1·529	−0·418	−1·105	−0·755	−1·393	−1·675	−0·696	0·984	1·917
	60	3·408	0·587	−1·647	−2·773	−3·001	−0·861	−0·796	0·166	1·742	2·919
	80	3·173	0·012	−2·374	−4·024	−5·096	−0·287	−0·213	0·179	0·833	1·412
	90	3·140	−0·064	−2·465	−4·205	−5·422	0·000	0·000	0·000	0·000	0·000
9	0	15·17	8·18	1·52	−1·45	−1·69	−1·80	−4·62	−7·90	−9·84	−9·85
	20	14·84	7·22	0·47	−1·74	−1·05	−1·60	−3·66	−5·57	−6·34	−6·19
	40	13·95	4·85	−2·18	−3·71	−2·06	−1·13	−1·67	−1·17	0·48	1·80
	60	12·86	2·30	−5·12	−7·72	−7·66	−0·60	−0·15	1·32	4·10	6·73
	80	12·15	0·71	−6·93	−11·12	−13·87	−0·18	0·14	0·87	2·20	3·67
	90	12·05	0·50	−7·22	−11·63	−14·75	0·00	0·00	0·00	0·00	0·00
10	0	−9·90	−3·98	0·43	1·18	1·06	−3·40	−1·87	4·65	11·10	12·35
	20	−9·75	−3·56	0·75	0·79	−0·20	−3·23	−2·20	2·86	7·86	9·06
	40	−9·30	−2·47	1·80	1·25	−0·70	−2·71	−2·64	−0·40	1·35	1·47
	60	−8·71	−1·23	3·31	3·96	3·31	−1·82	−2·27	−1·92	−2·55	−4·04
	80	−8·33	−0·46	4·41	6·74	8·49	−0·64	−0·89	−0·98	−1·59	−2·62
	90	−8·28	−0·36	4·57	7·18	9·36	0·00	0·00	0·00	0·00	0·00

TABLE XXIV c. VALUES OF ζ_1/H AND ζ_2/H FOR THE SEMIDIURNAL TIDE (K_2)

		ζ_1/H					ζ_2/H				
β	χ	$\theta = 20°$	40°	60°	80°	90°	20°	40°	60°	80°	90°
11	0°	−2·654	0·218	1·055	−0·378	−0·779	−4·446	−3·938	1·646	7·419	8·595
	20	−2·773	−0·021	1·024	−0·379	−1·250	−4·185	−3·899	0·627	5·278	6·436
	40	−2·842	−0·258	0·743	−0·720	−2·211	−3·393	−3·477	−1·038	1·056	1·447
	60	−2·776	−0·231	0·775	0·164	−0·497	−2·218	−2·471	−1·613	−1·584	−2·464
	80	−2·819	−0·266	1·454	2·682	3·798	−0·788	−0·925	−0·771	−1·072	−1·774
	90	−2·838	−0·284	1·607	3·198	4·649	0·000	0·000	0·000	0·000	0·000
12	0	1·38	4·80	3·93	−0·34	−1·29	−9·03	−8·39	1·73	11·88	13·70
	20	1·00	3·67	2·40	−2·43	−4·30	−8·43	−8·11	0·10	8·21	10·19
	40	0·14	1·37	−0·04	−4·39	−7·34	−6·79	−7·06	−2·76	0·47	0·98
	60	−0·66	−0·40	−0·74	−1·80	−2·76	−4·42	−4·90	−3·51	−4·20	−6·39
	80	−1·09	−1·15	−0·26	2·17	4·87	−1·55	−1·76	−1·52	−2·40	−4·04
	90	−1·14	−1·24	−0·15	2·82	6·22	0·00	0·00	0·00	0·00	0·00
13	0	−12·58	−16·90	−12·85	−3·40	−1·36	12·72	10·92	−4·47	−18·39	−20·27
	20	−11·07	−12·54	−6·74	4·22	8·08	11·92	11·05	−0·49	−11·14	−13·99
	40	−7·67	−3·84	3·12	12·95	19·12	9·72	10·65	6·58	4·93	4·98
	60	−4·48	2·78	6·64	7·70	9·05	6·44	8·10	8·55	13·97	20·72
	80	−2·76	5·72	5·88	−2·33	−10·60	2·27	3·04	3·71	7·09	11·89
	90	−2·54	6·06	5·65	−4·02	−14·10	0·00	0·00	0·00	0·00	0·00
14	0	−5·02	−5·74	−5·38	−4·28	−4·05	−0·70	−1·51	−2·44	−2·87	−2·74
	20	−4·43	−3·92	−2·51	−0·50	0·31	−0·59	−0·89	−0·84	−0·96	−1·50
	40	−3·13	−0·50	2·00	4·28	6·05	−0·34	0·37	2·09	3·79	3·96
	60	−1·90	1·93	3·47	2·95	3·14	−0·09	1·00	3·09	6·32	9·06
	80	−1·18	3·07	3·07	−0·76	−4·32	0·00	0·50	1·37	3·01	4·97
	90	−1·08	3·22	2·96	−1·40	−5·69	0·00	0·00	0·00	0·00	0·00
15	0	−5·37	−5·38	−8·55	−12·57	−13·21	−9·71	−10·86	−4·79	0·30	1·06
	20	−4·67	−2·86	−3·53	−5·03	−4·90	−8·92	−8·95	−2·21	2·30	1·17
	40	−3·08	1·68	4·06	5·20	7·21	−6·80	−4·78	2·80	7·68	7·35
	60	−1·63	4·36	5·87	4·17	4·28	−4·01	−1·33	4·79	10·85	14·98
	80	−0·93	5·13	4·50	−1·97	−7·91	−1·29	−0·08	2·17	5·05	8·27
	90	−0·85	5·20	4·21	−3·07	−10·25	0·00	0·00	0·00	0·00	0·00

TABLE XXIV D. VALUES OF ζ_1/H AND ζ_2/H FOR THE SEMIDIURNAL TIDE (K_2)

		ζ_1/H					ζ_2/H				
β	χ	$\theta = 20°$	40°	60°	80°	90°	20°	40°	60°	80°	90°
16	0°	2·12	0·26	9·79	23·19	25·49	21·26	21·46	6·18	−3·95	−4·56
	20	1·71	−2·06	3·19	11·57	12·92	19·57	18·27	3·07	−4·52	−2·37
	40	0·87	−5·41	−6·05	−4·69	−6·59	15·09	10·92	−3·32	−9·92	−7·90
	60	0·34	−6·07	−6·90	−4·42	−4·94	9·12	4·25	−6·10	−13·91	−18·51
	80	0·25	−5·54	−4·12	3·77	11·33	2·97	0·87	−2·81	−6·53	−10·70
	90	0·26	−5·44	−3·63	5·27	14·57	0·00	0·00	0·00	0·00	0·00
17	0	−0·607	−1·860	1·916	7·479	8·458	6·853	6·325	1·157	−1·360	−1·183
	20	−0·564	−2·122	0·278	4·015	4·723	6·314	5·444	0·534	−1·077	−0·055
	40	−0·389	−2·130	−1·774	−0·903	−1·343	4·880	3·392	−0·867	−2·085	−1·017
	60	−0·082	−1·402	−1·475	−1·002	−1·388	2·981	1·462	−1·520	−3·314	−4·284
	80	0·173	−0·770	−0·435	1·293	3·130	0·988	0·351	−0·697	−1·626	−2·731
	90	0·211	−0·684	−0·268	1·719	4·056	0·000	0·000	0·000	0·000	0·000
18	0	−0·868	−2·321	0·632	5·224	6·016	4·361	3·545	0·123	−0·769	−0·386
	20	−0·726	−2·224	−0·252	2·866	3·487	4·050	3·137	0·001	−0·302	0·652
	40	−0·356	−1·634	−1·209	−0·514	−0·812	3·159	2·059	−0·531	−0·728	0·262
	60	0·115	−0·611	−0·691	−0·621	−1·094	1·929	0·931	−0·846	−1·748	−2·257
	80	0·469	0·095	0·158	0·946	1·976	0·639	0·236	−0·376	−0·938	−1·674
	90	0·522	0·187	0·288	1·240	2·624	0·000	0·000	0·000	0·000	0·000
19	0	−0·895	−2·880	−0·009	4·750	5·570	3·283	2·049	−0·649	−0·471	0·104
	20	−0·709	−2·616	−0·689	2·530	3·180	3·074	1·942	−0·420	0·209	1·295
	40	−0·156	−1·583	−1·191	−0·601	−0·942	2·450	1·478	−0·344	0·052	1·124
	60	0·535	−0·157	−0·358	−0·596	−1·229	1·526	0·771	−0·519	−1·158	−1·619
	80	0·967	0·666	0·520	0·948	1·786	0·508	0·210	−0·271	−0·765	−1·500
	90	1·023	0·762	0·641	1·234	2·426	0·000	0·000	0·000	0·000	0·000
20	0	−0·951	−4·293	−0·782	5·467	6·538	2·648	0·586	−1·816	−0·238	0·644
	20	−0·584	−3·686	−1·356	2·803	3·676	2·509	0·874	−1·014	0·929	2·400
	40	0·307	−1·901	−1·550	−1·022	−1·580	2·078	1·131	−0·155	0·983	2·364
	60	1·331	0·252	−0·245	−0·911	−1·991	1·377	0·815	−0·221	−0·957	−1·691
	80	2·054	1·523	1·040	1·311	2·300	0·483	0·266	−0·171	−0·893	−1·976
	90	2·159	1·681	1·224	1·731	3·228	0·000	0·000	0·000	0·000	0·000